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Spatial Reasoning

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We carried out two experiments to investigate how people reason about the spatial relations among objects. The experiments were designed to test a theory of spatial inference based on mental models. The theory predicts that problems requiring only one model of the spatial layout to be constructed should be easier than those requiring more than one model to be constructed, even when the multiple-model problems have valid conclusions. A contrasting theory, based on rules of inference, predicts that problems based on fewer inferential steps should be easier than problems based on more steps. The first experiment held constant the number of inferential steps specified by the inference-rule theory, but varied the number of models required to make a valid response: the one-model problems were reliably easier than the problems requiring more than one model. The second experiment contrasted opposing predictions from the two theories, and once again the results supported the model-based theory.

In daily life, many inferences depend on relations. If, for example, a law asserts that dog owners must pay a tax, and Alicia owns a labrador, then one can readily infer that she must pay the tax. Strictly speaking, the inference is based on a missing premise that establishes the relation of class-inclusion between labradors and dogs. Ordinary individuals, unlike logicians, neither notice that the premise is missing nor categorize the inference as invalid on these grounds. They know that labradors are dogs and they automatically use this knowledge in drawing the conclusion. The relation of class inclusion is ubiquitous, perhaps because it gives rise to transitive inferences, and so it plays a central part in theories of semantic memory (e.g., Collins & Quillian, 1969; Rips, Shoben, & Smith, 1973; Miller & Johnson-Laird, 1976).

Comparative adjectives, such as "taller than," are another source of relations that yield transitive inferences. Thus, given the premises

Cathy is taller than Linda
Linda is taller than Mary,

it is easy to deduce that

Cathy is taller than Mary.

It is very much harder, however, to determine exactly how people make these so-called "three term series" inferences. Their introspective reports have little to say about underlying mental processes (Evans, 1989), and there has been considerable controversy about the correct interpretation of the various experimental results (see, e.g., Hunter, 1957; Huttenlocher, 1968; Clark, 1969; Potts, 1972; Griggs & Osterman, 1980; Sternberg, 1981; Richardson, 1987). For example, the difference between "negative-equatives" such as "x is not as good as y" and affirmative premises is consistent with various theories (see Evans, 1982 for a review). The problem of identifying the inferential mechanism is made still harder by the fact that people can use different strategies in solving these problems (Sternberg & Weil, 1980; Egan & Grimes-Farrow, 1982). These difficulties have left open the nature of the underlying processes of deductive reasoning, and it has been modelled in two quite distinct ways.

The first sort of theories are based on proof-theoretic methods, and they postu-

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late a mental logic consisting of formal rules of inference that are used to derive conclusions in a quasi-syntactic way (see, e.g., Inhelder & Piaget, 1958; Osherson, 1975; Braine, 1978; Rips, 1983). The first step according to such theories is to translate the premises into a language-like mental representation that makes their logical form explicit. Inference rules can then be applied to these representations in order to derive a conclusion. A rule must correspond to the forms to which it is applied (i.e., the premises, or assertions, derived for them). Whenever the premises fail to match the logical form of the rule, then the rule cannot be applied to them. If the conclusion is to be derived, it is mandatory either to find an alternative rule or to make a preliminary inference to an assertion that does have the required logical form.

The logical properties of relations can be captured by general schemas, such as the following one for transitivity (where "*R*" denotes a relation):

For any x, y, and z, if xRy and yRz, then xRz.

All transitive relations, such as "taller than," must be tagged in some way to indicate that this schema applies to them (BarHillel, 1967). Alternatively, each individual transitive relation can have its own meaning postulate, e.g.,

For any x, y, and z, if x is taller than y, and y is taller than z, then x is taller than z.

Experimenters have yet to determine whether such postulates are represented in the mind (but see, e.g., Cheng, Holyoak, Nisbett, & Oliver, 1986; Fodor, 1977; Kintsch, 1974).

The second sort of theories are based on model-theoretic methods, and they postulate semantic procedures for generating valid conclusions. Such theories applied to three-term series problems propose that people use their understanding of the premises to imagine the state of affairs they describe—they construct a scenario, image, or spatial array of the situation, which they then use as a basis for reasoning (see, e.g., De Soto, London, & Handel, 1965; Huttenlocher, 1968; Erickson, 1974; Sternberg, 1985).

An unfortunate fact about the study of three-term series problems is that it has not yielded any clear answer to the question of which of these two sorts of theory—the proof-theoretic or the model-theoretic—is correct. Moreover, no such answer seems likely to be forthcoming from studying these problems. They are so simple that the two theories appear to mimic one another in their predictions, and so it may be impossible to design a crucial experiment. A more promising domain concerns a richer sort of relational reasoning that has been hitherto neglected: spatial inference.

Previous studies of how people understand spatial descriptions, such as

The knife is in front of the vase.
The vase is on the left of the glass.
The glass is behind the dish.

have established that subjects tend to envisage such layouts as symmetric with approximately equal distances between adjacent objects (Ehrlich & Johnson-Laird, 1982) and that they can represent the content of such descriptions in two distinct ways, one closer in structure to the linguistic form of the description itself and the other to the structure of the state of affairs that is described (Mani & Johnson-Laird, 1982). In this paper, we will consider how people make inferences from spatial descriptions.

Spatial Reasoning by Rules or Models

Spatial reasoning is ubiquitous in our everyday interactions with the world: it underlies out ability to plan a route, to locate entities, and to envisage objects from descriptions of their arrangement. A simple sort of spatial inference depends on premises that describe a one-dimensional layout of objects, such as:

A is on the right of *B*
C is on the left of *B*
Hence, *A* is on the right of *C*.

A meaning postulate for making this inference is:

If *x* is on the right of *y*, and *z* is on the left of *y*, then *x* is on the right of *z*

and inference-rule systems containing such postulates have been proposed for human reasoning (e.g., Hagert, 1983; Ohlsson, 1981, 1984, 1988). A second sort of one-dimensional problem depends on premises such as:

B is on the right of *A*
C is on the left of *B*

which do not yield any valid conclusion about the relation between *A* and *C*. These premises fail to match any of the formal rules in the system and hence the response of no valid conclusion can be made (although, strictly speaking, a conjunction of the two premises could be inferred). An inference-rule theory might thus predict that people should find it easier to make an inference from the first problem than from the second problem.

The model-based theory makes the same prediction for these simple problems, but for different reasons. According to this theory, in order to understand the premise:

A is on the right of *B*,

it is necessary to grasp the meaning of the relational predicate, "on the right of." This knowledge, together with compositional principles that combine meanings according to syntactic relations, can be used to construct a representation of a specific situation that satisfies the premise.

A computer program implementing the theory builds up two-dimensional spatial arrays containing tokens in the positions described by such premises (see Johnson-Laird, 1983, Chap. 11). Given an initial premise of the form:

A is on the right of *B*,

it constructs an initial minimal array that satisfies the truth conditions of the premise:

B *A*

assuming a viewpoint such as an observer would have standing in front of the objects. The meaning of "on the right of" specifies an appropriate increment to one Cartesian coordinate while holding the other coordinate constant. The program checks each subsequent premise to determine whether it makes reference to any items already in the array. Given the second premise:

C is on the left of *B*,

it accordingly finds *B* in the model, and using its representation of the meaning of "on the left of," it inserts *C* into the array in an appropriate place:

C *B* *A*

A further assertion:

A is on the right of *C*

contains only referents that are already in the array, and so a procedure is called to verify the premise—again, using the meaning of "on the right of." The verification procedure returns the value "true" for the assertion because its truth conditions are satisfied by the items in the array. This evaluation, in turn, elicits a falsification procedure that searches for an alternative model of the previous premises that falsifies the current assertion. The procedure does not use inference rules sensitive to the logical form of the premises, but seeks to rearrange the model without violating the meaning of the premises. If there is such an alternative, then the current assertion provides new information by ruling out this possibility. In the present case, however, there is no such alternative and so, as the program indicates, the conclusion is *valid:* it must be true given that the previous premises are true. On this account the transitivity of a relation, such as "*x* is on the right of *y*" derives, not from an inference rule for transitivity, but from the meaning of the relation and the way that meaning is used in the construction of models.

Given the premises of the second problem:

B is on the right of *A*
C is on the left of *B*,

the program constructs the model:

C A B

A further assertion:

C is on the left of *A*

is once more, true in the model, and so the falsification procedure is triggered. In this case, however, the process of revising the model succeeds in constructing an alternative model of the premises:

A C B

which refutes the more recent assertion since the *C* is now on the right of the *A*. This model must be considered along with the previous one to determine whether there is any conclusion that holds over both of them—again, if there is such a conclusion, it too should be tested in the same way. In fact, there is no conclusion interrelating *A* and *C* that is true in both models and so the premises do not support a valid deduction.

For these one-dimensional problems, the number of models that a problem requires is confounded with the validity of the inferences: single-model problems support a valid conclusion, whereas multiple-model problems support no valid conclusion, and it is not possible to disentangle these two variables. However, other sorts of problems that describe a two-dimensional layout allow these variables to be examined separately. They also yield divergent predictions from the two sorts of theory.

Consider the following inference (Problem I):

1. *A* is on the right of *B*
2. *C* is on the left of *B*
3. *D* is in front of *C*
4. *E* is in front of *B*.
   Hence, *D* is on the left of *E*.

Hagert (1983) has proposed an ingenious inference-rule system for two-dimensional reasoning that uses the following sorts of one- and two-dimensional rules:

a. Left (x,y) & Front $(z, x) \rightarrow$ Left (front (z, x), y), where the right-hand side signifies "*z* is in front of *x*, all of which is on the left of *y*."

b. Left (x, y) & Front $(z, y) \rightarrow$ Left (x, front (z, y)), where the right-hand side signifies "*x* is on the left of *z* which is in front of *y*."

c. Left (x, y) & Left $(y, z) \rightarrow$ Left (x, left (y, z)).

d. Left $(x, y) \leftrightarrow$ Right (y, x).

e. Left (front (x, y), z) $\rightarrow$ Left (x, z) & Left (y, z) & Front (x, y).

f. Left (x, front (y, z)) $\rightarrow$ Left (x, y) & Left (x, z) & Front (y, z).

g. Left (x, left (y, z)) $\rightarrow$ Left (x, y) & Left (x, z) & Left (y, z).

h. Left $(x, y) \rightarrow -$ Right (x, y).

i. Right $(x, y) \rightarrow -$ Left (x, y).

The first premise in the problem is irrelevant, and the inference rules must be used to derive the relations between *D*, *C*, and *B*, and then those between *D*, *E*, and *B*, before there is sufficient information to use a rule to infer the relation between *D* and *E*. Thus, the proof proceeds by deriving the relation between *D*, *C*, and *B:*

5. *C* is on the left of *B* and *D* is in front of *C* [conjunction of premises 2 and 3].
6. *D* is in front of *C*, which is on the left of *B* [rule *a* to 5] which is equivalent to:
7. *D* is on the left of *B*, and *C* is on the

left of *B*, and *D* is in front of *C* [rule *e* to 6].

Now, the relation between *D*, *E*, and *B* can be derived:

8. *D* is on the left of *B* [conjunction elimination to 7].
9. *D* is on the left of *B* and *E* is in front of *B* [conjunction of 8 and premise 4].
10. *D* is on the left of *E* which is in front of *B* [rule *b* to 9] which is equivalent to:
11. *D* is on the left of *E*, and *D* is on the left of *B*, and *E* is in front of *B* [rule *f* to 10].
12. *D* is on the left of *E* [conjunction elimination to 11], which is the required conclusion.

The following problem (Problem II):

B is on the right of *A*
C is on the left of *B*
D is in front of *C*
E is in front of *B*.
Hence, *D* is on the left of *E*,

requires an identical derivation. The first premise is again irrelevant, and the remaining three are the same as those of Problem I. Hence, if people are using such rules, there should be no reliable difference in difficulty between Problem I and Problem II.

In contrast, the model-based theory predicts a difference in the difficulty of the two problems. The premises of Problem I yield the model:

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C B A
D E
```

The relation between *D* and *E* can be established in a way similar to the evaluation of the assertion:

D is on the left of *E*.

The procedure that revises models will not succeed in producing an alternative model that refutes this assertion, and so *D* must be on the left of *E*. Thus, this set of premises can be classified as a single-model problem that supports a valid conclusion.

The premises of Problem II, however, are consistent with at least two distinct models. One model is

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C A B
D   E
```

which supports the conclusion

D is on the left of *E*.

The falsification procedure succeeds in constructing an alternative model of the premises

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A C B
  D E
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but both of these models support the same conclusion, and there is no alternative model that falsifies it. Hence, Problem II is a multiple-model problem that supports a valid conclusion. The model-based theory predicts that a problem which requires more than one model will be harder than one which requires only one model, and so it predicts that Problem II should be harder than Problem I.

Some two-dimensional problems do not yield a valid answer, e.g., Problem III:

B is on the right of *A*
C is on the left of B
D is in front of *C*
E is in front of *A*.
What is the relation between *D* and *E?*

Both theories predict that these problems should be hardest of all. According to the inference-rule theory, they should be difficult because the response of "no valid conclusion" is forthcoming only after all possible derivations have been tried. According to the model-based theory, the premises of Problem III support two models:

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C A B     A C B
D E       E D
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in which there is no common relation between *D* and *E*. The theory predicts that this problem will be harder than the one-model Problem I. It also predicts that it will be harder than the multiple-model Problem

II that supports a valid conclusion. Where the multiple models support a valid conclusion, a reasoner who in fact constructs just one model will nevertheless draw the correct conclusion. However, where the multiple models do not support a valid conclusion, a reasoner who constructs just one model will make an erroneous inference, because the only way to appreciate that there is no valid conclusion is to consider more than one model of the premises.

Experiment 1

Because there had been no previous studies of two-dimensional spatial inferences, our principal aim was to find out whether people were capable of making such inferences. Evidence from three-term series problems suggested that subjects would have no difficulty with one-dimensional problems, and so we designed the experiment to compare the ease of such inferences with those based on two-dimensional layouts. We naturally expected that the two-dimensional problems would be harder than the one-dimensional ones.

Our second aim was to make a preliminary comparison between the inference-rule and the model-based theories. We therefore examined three sorts of two-dimensional problems: those that yield a valid conclusion based on one model (such as Problem I above), those that yield a valid conclusion based on more than one model (such as Problem II), and those that do not yield a valid conclusion from their multiple models (such as Problem III). The model-based theory predicts that the one-model problems should be easier than the multiple-model problems with a valid conclusion; the inference-rule theory makes no such prediction. Both theories predict that the problems lacking a valid conclusion should be hardest of all.

Method

Design and materials. The subjects acted as their own controls and each received five sorts of problem: one-dimensional problems with valid answers, one-dimensional problems with no valid answers, two-dimensional problems with valid answers, and which, according to the model-based theory, have one model, two-dimensional problems with valid answers and which, according to the model-based theory, have more than one model, and two-dimensional problems with no valid answers. The one-dimensional problems consisted of two premises and the two-dimensional problems consisted of four premises. The two-dimensional problems were in the following sort of orientation:

A	*B*	*C*
D		*E*

Each subject received five, six, or seven instances of each of the five types of problem, and each problem was presented with different lexical materials about various domains concerning familiar objects, such as cups and plates, drawn from the same semantic domain. These sets of objects were assigned to the problems at random in order to produce two sets of materials. Half the subjects received one set and half the subjects received the other set.

Procedure. The subjects were given the instructions by way of an example. They were told that the experimenter would read aloud a description of the layout of some objects, which they could imagine arranged on the tabletop in front of them. They were encouraged to listen attentively since the description would be read only once, although at a reasonable pace. They would then be asked about the location of two of the objects, e.g., "What is the relation between the cup and the plate?" In fact, the two items in the question were always the end-terms for the two-premise problems, and the pair of items (the *D* and the *E* in the examples above) that were not explicitly interrelated in any premise of the four-premise problems. Finally, the subjects were told that for some of the problems they might think there was not enough information to determine the relation between the two objects; in which case, they were to say so. Their responses were recorded on cassette recorder.

Subjects. The 15 subjects (11 women and 4 men) ranged in age from 19 to 53 years old. They were members of the APU subject panel, and they were paid £3 per hour for participating in the experiment, which lasted just under half an hour.

Results

The percentages of correct responses are presented in Table 1. Contrary to our expectations, the subjects coped as readily with the two-dimensional inferences (39% correct) as with the one-dimensional inferences (44% correct), and the difference between them was not reliable (Wilcoxon's $T = 45.5$, $n = 15$, $p > 0.05$). However, as Table 1 shows, the results corroborated the model-based prediction: there were more correct responses to the valid problems that required one model (61% correct) than to the valid problems that required more than one model (50% correct) and this difference was reliable (Wilcoxon's $T = 30.5$, $N = 15$, $p < 0.05$).

Finally the results corroborated the prediction common to the two theories: problems with a valid answer were easier than those with no valid answer. There were 59% correct responses overall to the problems with a valid conclusion, but there were only 18% correct responses to the problems without a valid conclusion (Wilcoxon's $T = 1$, $N = 15$, $p < 0.01$). This difference was reliable for both the one dimensional problems (Wilcoxon's $T = 1$, $N = 14$, $p < 0.01$), and for the two-dimensional problems (Wilcoxon's $T = 1$, $N = 15$, $p < 0.01$).

TABLE 1
THE PERCENTAGE OF CORRECT RESPONSES TO THE PROBLEMS IN EXPERIMENT 1

Descriptions	Single model (valid conclusion)	Multiple model (valid conclusion)	Multiple model (no valid conclusion)
Two-premise	69	—	19
Four-premise	61	50	18

Discussion

The experiment showed that people can reason about the spatial relations between objects and do not seem to find the two-dimensional spatial layouts any harder than the one-dimensional layouts. Problems with a valid answer are evidently easier than problems with no valid answer. This phenomenon is consistent with both the inference-rule theory and the model-based theory, but it would also have occurred if subjects were loath to respond that there is no definite answer. This explanation seems implausible, especially given so striking a difference in accuracy, because subjects do quite well with syllogisms that do not support a valid conclusion (see, e.g., Johnson-Laird & Bara, 1984; but cf. Rumain, Connell, & Braine, 1983).

The result that people make more correct inferences from one-model problems than from multiple-model problems corroborates the model-based theory. Because these two sorts of problem require identical formal derivations, the result cannot be predicted by the inference-rule theory. The premises used in the derivations for the two sorts of problem are the same. In Problem I, the irrelevant first premise could be used to derive, *en passant*, an ultimately useless inference about the relation between *A* and *C*. The derivation for Problem I would then contain an extra step interrelating them. The derivation for Problem II would be unaltered because the rules cannot interrelate *A* and *C*. But, this difference in the derivation lengths would predict incorrectly that Problem I should be harder than Problem II.

We have established that two-dimensional reasoning is a feasible task for examining theories of reasoning, and that the inference-rule theory that we have considered fails to predict an observed difference that is predicted by the model-based theory. A more stringent test, however, needs to pit the inference-rule and model-based theories directly against each other on problems where the inference-rule theory predicts a difference in one direction, and the model-based theory predicts a difference in the opposite direction. Our second experiment makes such a comparison between the two theories.

EXPERIMENT 2

In this experiment we examined three sorts of two-dimensional inferences. First, one-model problems of the following sort (Problem IV):

1. *A* is on the right of *B*
2. *C* is on the left of *B*
3. *D* is in front of *C*
4. *E* is in front of *A*
   What is the relation between *D* and *E*?

Unlike Problem I, there is no premise that directly asserts the relation between the pair of items to which *D* and *E* are directly related. Hence, the relation between them has to be inferred. The first two premises of the problem:

1. *A* is on the right of *B*
2. *C* is on the left of *B*

have to be used to make the following sort of derivation:

5. *B* is on the right of *C* [rule *d* to premise 2]
6. *A* is on the right of *B* and *B* is on the right of *C* [conjunction of 1 and 5]
7. *A* is on the right of *C* [transitivity rule to 6].

This inferred relation, together with the final two premises, now permits the same derivation as Problems I and II.

Second, multiple-model problems, such as Problem II above, that have a shorter derivation than Problem IV.

Third, multiple-model problems, such as Problem III above, that have no valid answers.

The model-based theory predicts that the one-model problems (IV) should be easier than the multiple-model problems with valid answers (II), whereas the inference-rule theory makes the opposite prediction. The one-model problems call for additional steps in their formal derivations over and above the derivations of the multiple-model problems and the inference-rule theory predicts that a conclusion requiring more inferential steps will be harder. Both theories predict, of course, that the problems with no valid answers should be hardest of all.

Method

Design. The subjects, acting as their own controls, carried out 18 inferences of three sorts: six one-model problems with valid answers, six multiple-model problems with valid answers, and six multiple-model problems without valid answers. Each problem was presented with different lexical materials. These lexical materials were assigned to the 18 problems at random in two different ways, and half the subjects received one assignment and half the subjects received the other assignment. The problems were presented in a different random order to each subject.

Materials and procedure. Four orientations formed the basis of the problems in each of three conditions:

(1)

A	*B*	*C*
D		*E*

(2)

E		*D*
A	*B*	*C*

(3)

D	*C*
	B
E	*A*

(4)

A	*E*
B	
C	*D*

The six problems in a condition consisted in one of each of these four orientations, and two extra problems in two of the four orientations, one vertical and one horizontal.

This variation, and the fact that the two items in the question, e.g.,

What is the relation between the *D* and the *E?*

did not occur in fixed positions in the premises, made it difficult for subjects to predict which items would occur in the question.

Each of the sets of five objects making up the contents of a problem was based on high frequency one-syllable words drawn from the same semantic domain. No two words within such a set began with the same initial letter. The procedure was identical to the first experiment, except that the subjects were given a four-term series problem as a preliminary practice problem. Furthermore, in order to try to improve performance, each problem was read out *twice* to the subject before they were asked the question about two of the objects.

Subjects. The 18 subjects (16 women and 2 men) ranged in age from 24 to 56 years old. They were members of the APU subject panel, and they were paid £3 per hour for participating in the experiment, which lasted for about half an hour.

Results and Discussion

The one-model problems (70% correct responses) were easier than the multiple-model problems with a valid conclusion (46% correct responses), which in turn were easier than the multiple-model problems without a valid conclusion (15% correct responses, Page's $L = 4.45$, $p < 0.05$; Page, 1963). The difference between the one-model problems and the multiple-model problems with valid answers was highly reliable (Wilcoxon's $T = 7$, $N = 14$, $p < 0.005$). This difference corroborates the model-based theory and runs counter to the inference-rule theory. Problems which require more models were harder than those which require only one model, but problems which require more inferential steps in their derivation were not harder than those which required fewer steps.

The subjects did not perform more accurately overall than those in the previous experiment, despite the repeated reading of the problems, and so their difficulties do not seem to be attributable to remembering the descriptions. Their performance did not improve with practice: they made as many correct inferences to their first six problems (47%) as they did to their next six problems (40%) and to their final six problems (48%, Page's $L = 3.82$, *n.s.*) Because the inferences depend on a two-dimensional layout, the chances of guessing the correct answer are approximately one in five: there are four obvious candidate spatial relations between the two objects, plus the response that there is no relation between them. Yet, there were only 15% correct responses to the multiple-model problems without a valid conclusion. These problems yielded 42% of responses in which the subjects interrelated the two objects on the dimension that was consistent with the premises (although the premises fail to support any definite relation on that dimension). Such responses are precisely what is to be expected if subjects are constructing only one of the possible models of the premises.

The errors on the other problems mainly consisted of conclusions that established the wrong relation. Where the subjects' conclusion is a relation orthogonal to the correct dimension, they have probably failed to build the correct model. Such errors should thus reflect the increasing difficulty of representing the premises: they occurred with 13% of one-model problems, but with 23% of multiple-model problems with a valid conclusion and 23% of multiple-model problems with no valid conclusion.

General Discussion

The results of both experiments show that it is easier to draw a valid spatial inference when a description corresponds to just a single layout as opposed to two or more distinct layouts. This phenomenon is readily explained if people naturally reason by imagining the state of affairs described in the premises, drawing a conclusion from such a mental model, and searching for alternative models that might refute that conclusion.

Could our instructions to imagine the objects' layout have caused our subjects to use an imagery-based strategy otherwise alien to them? We used this instruction—casually mentioned during the subjects' introduction to the task—to avoid any problems in the interpretation of "on the right of," "on the left of," and the other spatial terms. These expressions have two distinct senses: a "deictic" sense which depends on the speaker's point of view, and an "intrinsic" sense which depends on the parts of the objects (see, e.g., Miller & Johnson-Laird, 1976, Section 6.1). Only the former sense is guaranteed to support valid inferences of the sort used in the experiments (see Johnson-Laird, 1983), and the simplest way to ensure that the subjects made this interpretation was to tell them that the objects were being described from a particular point of view. It seems unlikely that this instruction could be powerful enough to lead the subjects to adopt a wholly unnatural reasoning strategy. Indeed, several authors lament the considerable difficulty of inducing reasoning strategies by explicit instructions (e.g., Dickstein, 1978), while others deny a significant role for imagery in reasoning (e.g., Richardson, 1987). The critical feature of the mental model theory concerns the structure of the representations used in reasoning—they should be similar to the structure of the world—rather than that they should be experienced as images.

Can experimental evidence really decide between model-based and inference-rule theories? There is little doubt that our two experiments fit the model-based theory rather than Hagert's (1983) inference-rule theory. But, could there be an alternative inference-rule theory that does account for our findings? One approach is worth describing because it illuminates the particular difficulties of such theories. It postulates such principles as:

If x is on the left of y,
and w is in front of x,
and z is in front of y,
then w is on the left of z.

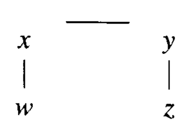
The first premise in Problems I and II is irrelevant, but the application of this rule to the remaining premises yields the conclusion directly:

The *D* is on the left of the *E*.

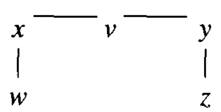
Because a large number of such rules would be necessary to deal with the full set of spatial relations, we can invoke abstract postulates of the same structure:

If x is related to y on one dimension,
and w is related to x on an orthogonal dimension,
and z has the same orthogonal relation to y,
then w is related to z in the same way as x is related to y.

This abstract rule has the advantage that it captures the content of a large number of specific rules, including the rule above because it applies to any configuration of the form



regardless of its rotation, or reflection. A system of such rules still makes the wrong predictions about relative difficulty. For example, the rule applies immediately to multiple-model problems, such as II, but fails to apply to one-model problems such as IV until the relation between x and y has been established. However, the rule might be revised so that it makes the required predictions (cf. Ohlsson, 1981, 1984, 1988). The resulting rule is more complex because it has to have the principle of transitivity built into it, i.e., the transitivity concerning x, v, and y, in the following sort of layout:



The revised rule has the form:

If x is related to v on one dimension
and v is related to y in the same way,
and w is related to x on an orthogonal dimension,
and z has the same orthogonal relation to y,
then w is related to z in the same way as x is related to y.

The cost of such a revision is severe, since the rule fails altogether to match the premises of the multiple-model Problem II, or even the premises of the one-model Problem I. The theory does not predict that such inferences will be harder; it predicts that they will be impossible. Yet, if both the original and the revised form of the rule are retained, then the theory ceases to predict the differences between one-model and multiple-model problems.

Inference-rule theories almost certainly have the computational power of universal Turing machines, and so there is unlikely to be any empirical phenomenon that cannot in principle be described within their framework. Our results do not refute inference-rule theories but rather present a challenge to them. To save the approach, it is necessary to devise a system of formal rules that explains why one-model problems are easier than multiple-model problems. The strategy behind this study was indeed to examine inferences where the number of mental models does not correlate with the number of inferential steps according to a particular inference-rule theory. Where the two theories diverge in this way, we have shown that performance follows the predictions of the model-based theory. We have used the same strategy with similar success in all the main domains of deductive inference, including propositional reasoning (Byrne, 1989; Johnson-Laird, Byrne, & Schaeken, 1989), and reasoning with single and multiple quantifiers (Johnson-Laird & Byrne, 1989; Johnson-Laird, Byrne, & Tabossi, in press). In each of these domains, the model-based theory relies, not on formal rules of inference, but on processes that construct models, formulate conclusions from them, and search for alternative models to serve as counterexamples. The logical consequences of words, like those denoting spatial relations, are nothing more than the emergent properties of their meanings within the system.

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REFERENCES

BAR-HILLEL, Y. (1967). Dictionaries and meaning rules. *Foundations of Language*, **3**, 409–414.

BRAINE, M. D. S. (1978). On the relation between the natural logic of reasoning and standard logic. *Psychological Review*, **85**, 1–21.

BYRNE, R. M. J. (1989). Suppressing valid inferences with conditionals. *Cognition*, **31**, 61–83.

CHENG, P. N., HOLYOAK, K. J., NISBETT, R. E., AND OLIVER, L. M. (1986). Pragmatic versus syntactic approaches to training deductive reasoning. *Cognitive Psychology*, **18**, 293–328.

CLARK, H. H. (1969). Linguistic processes in deductive reasoning. *Psychological Review*, **76**, 387–404.

COLLINS, A. M., AND QUILLIAN, M. R. (1969). Retrieval time from semantic memory. *Journal of Verbal Learning and Verbal Behaviour*, **8**, 240–247.

DE SOTO, C. B., LONDON, M., AND HANDEL, S. (1965). Social reasoning and spatial paralogic. *Journal of Personality and Social Psychology*, **2**, 513–521.

DICKSTEIN, L. S. (1978). Effects of instructions and premise order on errors in syllogistic reasoning. *Journal of Experimental Psychology: Human Learning and Memory*, **1**, 376–384.

EGAN, D. E., AND GRIMES-FARROW, D. D. (1982). Differences in mental representations spontaneously adopted for reasoning. *Memory and Cognition*, **10**, 297–307.

EHRLICH, K., AND JOHNSON-LAIRD, P. N. (1982). Spatial descriptions and referential continuity. *Journal of Verbal Learning and Verbal Behavior*, **21**, 296–306.

ERICKSON, J. R. (1974). A set analysis theory of behavior in formal syllogistic reasoning tasks. In

Solso, R. (Ed.) *Loyola Symposium on Cognition*, Vol. 2. Hillsdale, NJ: Erlbaum.

Evans, J. St. B. T. (1982). *The Psychology of Deductive Reasoning*. London: Routledge & Kegan Paul.

Evans, J. St. B. T. (1989). *Bias in human reasoning: Causes and consequences*. Hillsdale, NJ: Erlbaum.

Fodor, J. D. (1977). *Semantics: Theories of meaning in generative grammar*. New York: Crowell.

Griggs, R. A., and Osterman, L. J. (1980). Conversion errors in processing artificial set inclusions. *Quarterly Journal of Experimental Psychology*, **32**, 241–246.

Hagert, G. (1983). Unpublished manuscript, Uppsala Programming Methodology and Artificial Intelligence Laboratory, Uppsala University.

Hunter, I. M. L. (1957). The solving of three term series problems. *British Journal of Psychology*, **48**, 286–298.

Huttenlocher, J. (1968). Constructing spatial images: A strategy in reasoning. *Psychological Review*, **75**, 550–560.

Inhelder, B., and Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. London: Routledge & Kegan Paul.

Johnson-Laird, P. N. (1983). *Mental models: Towards a cognitive science of language, inference, and consciousness*. Cambridge: Cambridge University Press; Cambridge, MA: Harvard University Press.

Johnson-Laird, P. N., and Bara, B. G. (1984). Syllogistic inference. *Cognition*, **16**, 1–61.

Johnson-Laird, P. N., Byrne, R. M. J., and Schaeken, W. (1989). *Reasoning by model: The case of propositional inference*. Mimeo, MRC Applied Psychology Unit, Cambridge.

Johnson-Laird, P. N. and Byrne, R. M. J. (1989). *Only* reasoning. *Journal of Memory and Language*, **28**, 313–330.

Johnson-Laird, P. N., Byrne, R. M. J., and Tabossi, P. (in press). Reasoning by model: The case of multiple quantification. *Psychological Review*.

Kintsch, W. (1974). *The representation of meaning in memory*. Hillsdale, NJ: Erlbaum.

Mani, K., and Johnson-Laird, P. N. (1982). The mental representation of spatial descriptions. *Memory and Cognition*, **10**, 181–187.

Miller, G. A., and Johnson-Laird, P. N. (1976). *Language and perception*. Cambridge: Cambridge University Press; Cambridge, MA: Harvard University Press.

Ohlsson, S. (1981). *Model-based* vs. *propositional inference in a spatial reasoning task*. Unpublished paper, Department of Psychology, University of Stockholm.

Ohlsson, S. (1984). Induced strategy shifts in spatial reasoning. *Acta Psychologica*, **57**, 46–67.

Ohlsson, S. (1988). *Trace analysis and spatial reasoning* (Tech. Rep. No. KUL-87-02). Learning Research and Development Center, University of Pittsburgh.

Osherson, D. (1975). Logic and models of logical thinking. In R. J. Falmagne (Ed.) *Reasoning: Representation and process in children and adults*. Hillsdale, NJ: Erlbaum.

Page, E. B. (1963). Ordered hypotheses for multiple treatments: A significance test for linear ranks. *Journal of the American Statistical Association*, **58**, 216–230.

Potts, G. R. (1972). Information processing strategies used in the encoding of linear orderings. *Journal of Verbal Learning and Verbal Behaviour*, **11**, 727–740.

Richardson, J. T. E. (1987). The role of mental imagery in models of transitive inference. *British Journal of Psychology*, **78**, 189–203.

Rips, L. J. (1983). Cognitive processes in propositional reasoning. *Psychological Review*, **90**, 38–71.

Rips, L. J., Shoben, E. J., and Smith, E. E. (1973). Semantic distance and the verification of semantic relations. *Journal of Verbal Learning and Verbal Behaviour*, **14**, 180–195.

Rumain, B., Connell, J., and Braine, M. D. S. (1983). Conversational comprehension processes are responsible for reasoning fallacies in children as well as adults: IF is not the Biconditional. *Developmental Psychology*, **19**, 471–481.

Sternberg, R. J. (1981). Representation and process in linear syllogistic reasoning. *Journal of Experimental Psychology: General*, **109**, 119–158.

Sternberg, R. J. (1985). *Beyond I.Q.* Cambridge: Cambridge University Press.

Sternberg, R. J., and Weil, E. M. (1980). An aptitude-strategy interaction in linear syllogistic reasoning. *Journal of Educational Psychology*, **72**, 226–239.

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