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The Truth of Conditional Assertions

Geoffrey P. Goodwin, P. N. Johnson-Lairdb,c

^aDepartment of Psychology, University of Pennsylvania ^bDepartment of Psychology, Princeton University ^cDepartment of Psychology, New York University

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Abstract

Given a basic conditional of the form, *If A then C*, individuals usually list three cases as possible: *A and C*, *not-A and not-C*, *not-A and C*. This result corroborates the theory of mental models. By contrast, individuals often judge that the conditional is *true* only in the case of *A and C*, and that cases of *not-A* are irrelevant to its truth or falsity. This result corroborates other theories of conditionals. To resolve the discrepancy, we devised two new tasks: the "collective" truth task, in which participants judged whether sets of assertions about a specific individual, such as: *If A then C*, *not-A*, *C*, could all be true at the same time; and one in which participants judged the truth of conditional predictions about specific future events. The results consistently matched the three possibilities, thereby corroborating the model theory. They also showed a massive violation of the probability calculus in estimates of the probabilities of the four cases in the partition of conditionals (*A and C*, *A and not-C*, *not-A and C*, and *not-A and not-C*), which summed to over 200%.

Keywords: Conditionals; Mental models; Suppositional theory; Defective truth table; de Finetti truth table: Possibilities

1. Introduction

Conditional assertions, such as "If the Republicans win, then the market will rally" are a source of many puzzles. One is the conflict between the cases in which naive individuals — those who have not studied logic — consider them to be true, and the cases that they enumerate as possible given the truth of the conditional. The task that psychologists take to be fundamental is reflected in their theories of the meaning of conditionals (e.g., Evans & Over, 2004; Johnson-Laird & Byrne, 2002). The present paper aims to elucidate

Correspondence should be sent to Geoffrey P. Goodwin, Department of Psychology, University of Pennsylvania, 3720 Walnut Street, Philadelphia, PA 19104-6241. E-mail: ggoodwin@psych.upenn.edu

the conflicting results of the two tasks. Its plan is to outline these results, to describe two new tasks designed to resolve the conflict, and to report the results of four experiments using these tasks. Finally, the paper considers the implications of their results for current theories of conditionals. Its focus is on conditionals that refer to specific events, outcomes, or individuals, and not those that make general claims as in conditional rules or principles (see Goodwin, 2014; Sperber, Cara, & Girotto, 1995).

In the "truth-table" task, individuals evaluate a conditional, such as "If there's an A on the left then there is a 7 on the right," in light of a piece of conjunctive evidence, such as a card showing: A \mid 7 (e.g., Johnson-Laird & Tagart, 1969), where the vertical line demarcates the two halves of the card. They tend to evaluate the conditional as true in the case in which both of its clauses are true. Likewise, they tend to evaluate the conditional as false in the case in which the *if*-clause is true and the *then*-clause is false, for example, A \mid 3. But they tend to judge any other case, in which the *if*-clause is false such as: B \mid 7, as "Irrelevant" to the truth value of the conditional. Many investigators cite these and other similar results (e.g., Evans, 1972) in arguing that conditionals have a "defective" truth table (e.g., Evans & Over, 2004, p. 36), which is also known as the de Finetti (1974) truth table. Table 1 presents this truth table, which lays out the truth values of *If A then C* as a function of each possible case based on the truth or falsity of *A*, and the truth or falsity of *C*. The table uses the negation of a clause, such as *not-A*, to represent that *A* is false. As the table shows, the conditional has no truth value in the two cases in which its *if*-clause is false.

There are at least three ways to interpret the de Finetti truth table. First, "no truth value" can mean that the truth function returns no value whatsoever — it is void, just as division by zero is. This interpretation is perhaps the most prevalent in the recent psychological literature. An instance of it occurs in Baratgin, Over, and Politzer (2013), who wrote about conditional bets, "When A turns out to be false, no indicative assertion or bet is made. The assertion and bet are *void*: the indicative conditional *if A then C* is neither true nor false, and the bet on it is neither won nor lost" (p. 309; see also Politzer, Over, & Baratgin, 2010). Similarly, Pfeifer (2013) wrote that "... this sentence is crucial for explaining the semantics of the conditional event: *If A, then C* is *void*, if A is false" (p. 332). Most directly, Politzer et al. (2010) wrote that "The *not-A* cases, [i.e., false

Table 1
The "defective" truth table for a conditional *If A then C*, the contrasting truth table for material implication, *A materially implies C*, and the typical responses when individuals list what is possible and what is impossible given that *If A then C* is true

Cases, i.e., Contingencies in the World	The Defective Truth Table for <i>If A then C</i>	The Truth Table for <i>A materially implies C</i>	Responses to <i>If A</i> then C in Listing Possibilities
A and C	True	True	Possible
A and not-C	False	False	Not possible
Not-A and C	No truth value	True	Possible
Not-A and not-C	No truth value	True	Possible

antecedent cases] are irrelevant to the question [of whether the conditional is true] because the conditional is neither true nor false" (p. 176). To explain this point, they wrote that "when ... not-A is known, the indicative (1) [i.e., If A then B] is neither true nor false in probability theories and can also be termed void" (p. 174).

Second, "no truth value" can mean that the truth value has yet to be determined, so it will become either true or false. Perhaps de Finetti (1936/1995, p. 182) had this view in mind when he wrote that it is akin to an entry of "sex unknown" in a statistical survey. This interpretation is also apparently endorsed by Pfeifer (2013), who wrote immediately following the quote above that "This means that the truth value of the conditional is undetermined if the corresponding antecedent is false" (p. 332), thereby appearing to conflate "voidness" and "unknown" (or "undetermined").

Third, "no truth value" can mean that the value is neither true nor false, but some third value such as a conditional probability (Jeffrey, 1991). What the three interpretations have in common is that a conditional is neither true nor false when evidence shows that its *if*-clause is false.

Another sort of task yields very different results. In the "possibilities" task, participants are invited to accept a conditional of the form, *If A then C*, which has sensible contents. Their task is then to list what is possible (and in some studies what is impossible). They tend to list the following cases as possible:

Table 1 lists these possibilities, which occur with various sorts of conditional (see, e.g., Barrouillet & Lecas, 1998; Johnson-Laird & Savary, 1995; Lecas & Barrouillet, 1999; Quelhas, Johnson-Laird, & Juhos, 2010).

The conflicting results from the two tasks are contrary to the intuition that any case which an assertion refers to as possible should also be one in which the assertion is true. The discrepancy also raises a further question: Which task reveals the real meaning of conditionals? Psychologists have had three main answers to the question depending on their theoretical proclivities.

The first reaction is that the truth-table task is veridical. Many theorists accordingly accept the de Finetti truth table as stating the meaning of conditionals, that is, the cases in which they are true (e.g., Evans, 2012; Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011; Oaksford & Chater, 2007, p. 109; Politzer et al., 2010). It is a cornerstone of the "new paradigm" in studies of reasoning, which introduces probabilistic considerations into deductions (for a review, see Elqayam, Bonnefon, & Over, 2013; for a contrasting critical perspective, see Johnson-Laird, Khemlani, & Goodwin, 2015).

The second reaction is that the possibilities task is veridical. According to the theory of mental models — henceforth, the "model theory" (Johnson-Laird & Byrne, 2002)—the meanings of conditionals and other compound assertions refer to sets of possibilities. A "basic" conditional, *If A then C*, is one whose interpretation is unaffected by the

content of its individual clauses, their referents, or knowledge. It has two *mental* models, which are represented initially:

Α ...

The first model represents the possibility in which A and C hold, and the second model represents implicitly the possibilities in which A does not hold. These mental models yield intuitions and explain many phenomena in reasoning, but people can deliberate and flesh out mental models into *fully explicit* models. The mental models of a basic conditional are fleshed out into fully explicit models in the following order (see e.g., Khemlani, Orenes, & Johnson-Laird, 2014):

A C (the original explicit mental model)

Not-A Not-C (a possibility that holds for conditionals and for biconditionals)

Not-A C (a possibility that holds only for conditionals)

The same trend occurs developmentally in children's interpretations of conditionals, though the capacity of working memory is a better predictor than age (Barrouillet & Lecas, 1999).

According to the model theory, the content of a conditional and general knowledge can modulate its interpretation. They can prevent the construction of models of certain possibilities, for example, yielding a biconditional interpretation equivalent to if, and only if, A then C, in a case such as: If it rained, then it poured (e.g., Juhos, Quelhas, & Johnson-Laird, 2012). They can also establish temporal and other relations between A and C (Johnson-Laird & Byrne, 2002; Juhos et al., 2012). The algorithm for interpretation must, therefore, take content into account, but its output for some conditionals, such as, If the animal is a dog then it has four legs, is the set of three possibilities shown in Table 1. These possibilities correspond to the "True" cases in material implication, a logical connective with a truth table also shown in Table 1. Many critics of the model theory have accordingly supposed that the model theory postulates that basic conditionals are material implications. In fact, it makes no such assumption, and we will return to this issue in the General Discussion. Of course, for other conditionals, the resulting possibilities correspond to a different truth table, or to no truth table at all, as when interpretation introduces a temporal relation. Hence, interpretation cannot be truth functional (Johnson-Laird & Byrne, 2002). This claim is corroborated in the finding that people judge the not-A cases as possible not only for conditionals that are true a priori, but also for conditionals that are false a priori (Quelhas, Rasga, & Johnson-Laird, 2017). For example, the case of not-A and C is judged possible both for the true conditional, If Pat is reading the article then Pat is alive, and also for the false conditional, If Pat is reading the article then Pat is dead. Judgments that the not-A cases are possible for the false conditional rule out material implication, according to which such cases are false (see Quelhas et al., 2017).

The third reaction to the discrepancy between the truth-table task and the possibilities task embraces both sets of results. Barrouillet, Gauffroy, and Lecas (2008) distinguish between reasoning about possibilities given the truth of a conditional and evaluating the truth of a conditional given various states of affairs. They argue that the model theory of conditionals accounts for the possibilities task but needs to be revised in order to deal with the truth task. As part of this revision, they claim that there is a difference in the epistemic status of initial and fleshed-out models of a conditional. In the case of a conditional such as, *If A then C*, only its initial model, *A C*, is capable of making the conditional true, whereas the false *if-clause* cases, *not-A C*, and *not-A not-C*, are compatible with the conditional, but do not make it true. Furthermore, this difference in epistemic status accounts for why the initial model is preferentially represented. These authors contend that this hybrid theory is able to account for results from both the truth-table and possibilities tasks.

In the present paper, we take a similar view but argue in addition that the impasse can be resolved by taking a new, more critical, look at the standard truth task. The core result from this task that appeals to proponents of the defective truth table is that a conditional has no truth value when its if-clause is false. But is this always the case? In order to try to answer the question, we used two new experimental procedures. These two new tasks were designed to obviate several problems with the standard truth-table task that prevent a clear interpretation of its results. In the first task, we modified the truth-table task in order to test whether the defective truth table applies to conditionals that elicit thinking about alternative possibilities, and that decrease the focus on the main clause, the thenclause, of conditionals. In the second task, the participants have to decide whether or not a set of assertions is consistent. Naive individuals have difficulty in answering this framing of the question: "Could all of these assertions be consistent with one another?" and so we translated it into an equivalent question: "Could all of these assertions be true at the same time?" (see Goodwin & Johnson-Laird, 2010; Johnson-Laird, Legrenzi, Girotto, & Legrenzi, 2000; Johnson-Laird, Lotstein, & Byrne, 2012). This "collective" truth task asks only for judgments about the possible truth of conditionals. It does not call for an individual assessment of each possibility to which a conditional refers, and so it circumvents the potential issue of inferring a definite truth value from a single piece of evidence (see Schroyens, 2010a). It also circumvents a construal of the task as applying only to the main clause of a conditional, because the participants evaluate a set of assertions in which only one is a conditional, thereby defocusing attention from the conditional alone. We expand on the description of these issues presently. We investigated our modified truth-table task in Experiment 1, before turning to the collective truth task in Experiments 2-4. All of the conditionals in these experiments were ones that referred to specific events or outcomes or to the properties of specific individuals.

2. Experiment 1

In a modified version of the truth-table task, the experiment used trials of the following sort:

John is deciding whether to fire one of his two employees, Charlie or Annie, and possibly both of them.

In the end, John fired Annie.

A colleague of John's had predicted:

John will fire Charlie, if not Annie.

Was the colleague's prediction true, false, or irrelevant, in light of what happened? What do you think?

The task has two important features for our purposes. First, it frames the problem with an initial inclusive disjunction and with a conditional that has a negative *if*-clause, *C if not-A*. So, when individuals represent the conditional, they should tend to think of more than one possibility (see e.g., Byrne & Johnson-Laird, 1992), including the one represented in the mental models of the conditional, *C and not-A*, but also the one in which the *if*-clause is false, namely, *A*. They should therefore build at least these models of the people whom John fired:

Charlie Not Annie Annie

They may even envisage the contingency in which both were fired (Charlie Annie). This modification is important, because we suspect that existing results with the standard truth task may be brought about in part by participants failing to represent all relevant possibilities.

Second, in this revised task, because of the *C if not-A* formulation, "if" precedes an ellipsis (a partial clause), such as "if not Annie," rather than a complete clause. This formulation should reduce any bias to take the *if*-clause for granted, and to focus only on the *then*-clause. This may obviate another factor that accounts for the results of the standard truth task. In the standard version of the truth task, a bias may exist to carry out the task on the main clause of the conditional because doing so is cognitively less demanding (see e.g., Girotto & Johnson-Laird, 2004; Johnson-Laird & Byrne, 2002). Hence, if an experiment poses the question:

In what cases is it true that if A then C?

participants may paraphrase the task more simply as:

If A then in what cases is it true that C?

In other words, they may engage in a process of "attribute substitution," whereby they implicitly replace a difficult question with one that is easier to answer (Kahneman & Frederick, 2002). To answer the second question, participants presuppose that A is the case, and merely evaluate the truth or falsity of C from the cases presented to them. Accordingly, cases in which A does not hold are genuinely irrelevant to the task. The result, of course, is a defective truth table. Proponents of the defective truth table might argue that

this process accurately captures the meaning of the conditional. Yet, it may be that when individuals do not take the *if*-clause for granted, their evaluations no longer yield the defective truth table; that is, in part, what we aimed to test. We used ellipses with a negative phrase, for example, "John will fire Charlie, if not Annie," rather than with an affirmative phrase, for example, "John will fire Charlie, if Annie," which is odd in our dialect.

The experiment examined judgments of conditionals in light of the four possible contingencies of evidence (the partition) for conditionals of the form: C if not-A. The evidence C & not-A satisfies both clauses of the conditional, and so participants should respond "true." The evidence not-C & not-A satisfies the if-clause but not the then-clause, and so participants should respond "False." And the two sorts of evidence in which A occurs correspond to false if-clauses, and so the defective truth table predicts that participants should respond "Irrelevant." But, if the task elicits evaluations closer to the model theory, then the participants should tend to judge the conditional as "True" in these cases. The overall trend of "True" evaluations should follow the same order in which mental models tend to be fleshed out as fully explicit models: C & not-A, in which both clauses of the conditional are true, should yield more "True" responses than not-C & A, in which both clauses are false, and the latter should in turn yield more "True" responses than C & A, in which the if-clause is false and the then-clause is true.

2.1. Method

2.1.1. Participants

Eighty-eight participants in the United States began the experiment on a web survey program. Eight participants did not complete the experiment, and so their partial data were removed from the analyses, yielding 80 participants (63 male, 37 female) in the experiment proper.

2.1.2. Design and materials

The participants acted as their own controls and made judgments of the truth or falsity of conditionals presented in the form: *C if not-A*. For each problem, the conditional asserted a prediction about a relation between two future events and was presented after a definitive set of facts regarding those events (see the example above). The participants then judged whether the conditional prediction was true, false, or irrelevant, in light of the known facts. Each conditional, *C if not-A*, occurred on separate trials with four sorts of evidence; for the example above:

John fired Charlie: (C and not-A)
John fired Annie: (not-C and A)
John fired both Charlie and Annie: (C and A)
John fired neither Charlie nor Annie: (not-C and not-A)

The first sort of evidence did not state explicitly that Annie had *not* been fired, because it is implied in the overall context, and the assertion is simpler and more natural without

this explicit stipulation. Likewise, for the same reasons, the second sort of evidence did not state explicitly that Charlie had not been fired.

There were five sets of contents in total, and their framings and conditional predictions were as follows:

- 1. John is deciding whether to fire one of his two employees, Arthur or Elaine, and possibly both of them.
 - Conditional prediction: John will fire Arthur, if not Elaine.
- 2. Alan is deciding whether to buy a belt or a tie, and possibly both of them. *Conditional prediction*: Alan will buy the tie if not the belt.
- 3. Mary is deciding whether to order a coffee or a dessert, and possibly both of them. *Conditional prediction*: Mary will order a coffee if not a dessert.
- 4. Catherine (a triathlete) is deciding whether to go for a run or a swim, and possibly both of them.
 - Conditional prediction: Catherine will go for a run if not a swim.
- 5. A military general is deciding whether to order a ground invasion or an air strike.

 Conditional prediction: The general will order a ground invasion if not an air strike.

Each conditional was presented with all four possible contingencies outlined above, leading to a total of 20 experimental problems. Four additional problems were constructed, each with unique contents, and were presented with the *not-C and not-A* contingency. They were included in order to balance out the number of correct, "No," responses across the experiment. Each participant completed the full set of 24 problems in a different random order. When the participants had finished these problems, they completed four subsidiary problems in which they judged what possibilities arose from four unrelated conditionals. The results of this task are not pertinent to the present investigation, and so we do not report them.

2.1.3. Procedure

The Qualtrics web survey program administered the experiment, and each problem was presented on a new onscreen page. At the end of the experiment, participants provided some basic demographic information and were thanked and debriefed. The program collected no other data.

2.2. Results

Table 2 presents the percentages of the three sorts of evaluation of the conditionals as true, false, or irrelevant, depending on the nature of the evidence. The critical question for the defective truth table is whether participants tended to evaluate conditionals as irrelevant when their *if*-clauses were false. As Table 2 shows, however, the majority of participants evaluated them as true in such cases. When both the *if*-clause and the *then*-clause were false, they evaluated the conditional as true for 56% of trials. This percentage was reliably greater than chance (33.33%; Wilcoxon test, z = 4.33, p < .001), and

Table 2
The percentages of judgments that conditional predictions were true, false, or irrelevant, for the four possible
sorts of evidence in Experiment 1

	Judgment of the Conditional Prediction		
Evidence in Relation to Conditional	True	False	Irrelevant
Both clauses of conditional true	91	5	4
Both clauses of conditional false	56	27	17
If-clause false & then-clause true	56	33	11
If-clause true & then-clause false	3	89	8

reliably greater than the percentages of either "Irrelevant" (17%) or "False" (27%) evaluations (Wilcoxon tests, z's = 4.90 and 3.05, ps < .001 and = .002, respectively). Similarly, when the if-clause was false and the then-clause was true, the participants evaluated the conditional as true on 56% of the trials, a percentage that was again reliably greater than chance (Wilcoxon test, z = 4.44, p < .001), and also greater than the percentages of "Irrelevant" (12%) or "False" (33%) evaluations (Wilcoxon tests, z's = 5.35 and 2.71, ps < .001 and = .007, respectively). These patterns also occurred for each of the five individual conditionals. The most frequent evaluation when the if-clause was false was "True" in each case. For the false if-clause cases, out of 20 total comparisons comparing "True" with the "False" and "Irrelevant" evaluations, 18 were significant (using Sign tests), while the remaining two showed the predicted trend.

The model theory's predicted stochastic trend of "True" depending on the nature of the evidence was corroborated: when both clauses were true, there were more "True" evaluations (92%) than when both clauses were false (56%), which in turn yielded fractionally more "True" evaluations than when the *if*-clause was false and the *then*-clause was true (56%). Despite the negligible difference between the latter two cases, the predicted trend in rank orders was highly reliable, Page's L = 1025.50, z = 5.18, z = p < .001. Finally, as in all previous studies, when the *if*-clause was true and the *then*-clause was false, most evaluations (89%) of the conditional were "False" — a proportion much greater than chance (Wilcoxon test, z = 7.99, p < .001). This result shows that there was no general bias to evaluate conditionals as true.

2.3. Discussion

The new sort of truth-table task, which emphasized alternative possibilities and defocused participants from the main clause of the conditionals, produced evidence contrary to the defective truth table. As Table 2 shows, "Irrelevant" was overall the least frequent evaluation. The participants most often judged that the conditional was true when its *if*-clause was false. For example, the evidence:

Alan bought the tie and the belt

led to "True" evaluations of the conditional prediction:

Alan will buy the tie if not the belt.

These results refute the defective truth table as a general account of the truth conditions of conditionals. Its proponents, however, could argue that such conditionals with elliptical negatives rather than complete *if*-clauses are unusual, and so the present results may not generalize to more frequent conditionals in everyday life (Gernot Kleiter, personal communication, July 9, 2014). We agree. Yet the theories motivating the defective truth table offer no immediate explanation for the present results (pace, e.g., Evans & Over, 2004, p. 36). In contrast, the model theory motivated the present version of the truth-table task, and its results corroborated the theory.

However, there remains something odd about evaluating the truth value of a compound assertion, such as a conditional, in light of a single piece of evidence (Schroyens, 2010a). Indeed, as Barrouillet et al. (2008) showed, some adult reasoners resist the tendency to regard even the conjunction of the true *if*-clause and the true *then*-clause as providing decisive confirmation of a conditional. Schroyens (2010a) argued that this problem invalidates the truth-table task as providing diagnostic evidence for the ordinary semantics of the conditional. While his arguments apply to conditionals that state general rules, they may also apply to the conditionals in this study, which make predictions about alternative definitive events (for more on this distinction, see Goodwin, 2014; Sperber et al., 1995). Hence, perhaps the "collective" truth task, to which we now turn, casts a clearer light on the conditions in which conditionals are true.

3. Experiment 2

In the collective truth task, participants have to assess whether or not a set of assertions could all be true at the same time. The task is intimately related to valid deduction, but it differs in a crucial way. If reasoners use rules of inference to deduce a valid conclusion, they cannot use them directly to deduce that a set of assertions can all be true at the same time. The only general way to use rules of inference in this task is to try to prove the negation of one assertion in the set from the remaining assertions. If successful, then the set of assertions is not consistent; but if no such proof exists, then the set of assertions is consistent. This procedure is implausible for naive reasoners, and as far as we know, no psychologist has ever defended it. The model theory provides a simpler procedure. Consider a set of assertions of this sort:

If A then C A C

The mental models of the conditional:

A C

establish at once that all three assertions can be true together, because they all hold in the only mental model with explicit content. Now, consider three assertions of this sort:

> If A then C Not-A Not-C

This set of assertions does not hold in the initial mental model of the conditional (which contains only A and C), and so some individuals may respond, "No, the set is not consistent." But, others may flesh out the models of the conditional into fully explicit ones:

The second model here represents a possibility in which all three assertions hold, and so individuals who succeed in fleshing out the models in this way will make the correct response, "Yes, the set is consistent."

Conditionals that are not framed explicitly as biconditionals (i.e., of the form: *if and only if*), are nonetheless often interpreted as biconditionals. One reason is that biconditionals have only two fully explicit models, and so it is easier to make such an interpretation; another reason is that the semantic relation between their constituent clauses may yield a biconditional interpretation. For instance, given that dogs have four legs, a conditional about a particular animal, such as "If the animal has four legs then it is a dog," asserts a biconditional; that is, when its *if*-clause is false so too is its *then*-clause.

The model theory accordingly makes three predictions about the collective truth task. First, individuals should judge that a basic conditional can be true when its *if*-clause is false. Hence, they should judge that the assertions in each of the following sets 1, 2, and 3 could all be true at the same time, but not those in set 4:

1.	2.	3.	4.
If A then C			
A	Not-A	Not-A	A
C	Not-C	C	Not-C

With a biconditional interpretation, they should judge that the assertions in each of sets 1 and 2 could be true at the same time, but not those in sets 3 and 4.

Second, the order in which mental models are fleshed out as fully explicit should predict both the likelihood that a set is endorsed as collectively true and the latency of that judgment. This order predicts the following trends in endorsements of collective truth and their latencies: Set 1 should be endorsed more often and faster than Set 2, which in turn should be endorsed more often and faster than Set 3.

Third, conditionals that express a contingent relation as opposed to a known set-theoretic relation corresponding either to a basic or biconditional interpretation should tend to

yield a less clear-cut interpretation of one sort or the other. We amplify the motivation for this prediction in outlining the experimental materials.

3.1. Method

3.1.1. Participants

One hundred and seven participants in the United States began the experiment on Amazon.com's Mechanical Turk system in exchange for a small monetary payment. Six participants did not complete the experiment, and so their partial data were removed from the analyses, yielding 101 participants (44 male, 57 female) in the experiment proper.

3.1.2. Materials

Each problem consisted of three assertions, and the first assertion was always a conditional. The remaining two assertions in the set were drawn from the conditional's partition, that is, all four possible combinations of the affirmation and the negation of the two constituent clauses in the conditional. Hence, there were four grammatical forms of problem, as shown in the numbered sets above.

We devised four different sorts of conditional assertion, which differed in the nature of the semantic relation between their *if*-clause and their *then*-clause. The first sort implied a basic interpretation, for example, is as follows:

If the animal is a dog then it has four legs

where the framing of the problem made clear that the conditional referred to a single, unique, animal. The set to which the *if*-clause refers, *dogs*, is a proper subset of the set to which the *then*-clause refers, *has four legs*, and so the assertion refers to three possibilities: The animal is a dog and has four legs, it is not a dog and does not have four legs, it is not a dog and has four legs. The second sort of assertion expressed the converse relation, again about a single, unique animal:

If the animal has four legs then it is a dog.

The set in the *if*-clause is a superset of the set referred to in the *then*-clause, and so it should yield a biconditional interpretation, referring only to two possibilities: the animal has four legs and is a dog, and it does not have four legs and is not a dog. The putative third possibility (the animal doesn't have four legs and is a dog) is ruled out by the knowledge that dogs have four legs, at least normally. The third sort of assertion expressed a contingent relation between the sets referred to in the *if*-cause and in the *then*-clause, for example:

If the animal is a bird then it has black eyes.

The fourth sort of assertion was the converse of the third:

If the animal has black eyes then it is a bird.

We refer to both the third and fourth sorts of assertion as having a *contingent* interpretation, because they do not express a known subset or superset relation between the sets referred to in their two clauses. Given an experimental context in which participants make both basic conditional and biconditional interpretations, previous studies suggest that participants should tend to make more conditional interpretations of contingent conditionals, but should be open to biconditional interpretations too (see e.g., Johnson-Laird, Byrne, & Schaeken, 1992).

We devised four different sets of contents for each of the four sorts of conditional. The materials used one-syllable animal categories (e.g., dog, bird, etc.) and simple, two-word physical attributes (e.g., blue fins, two ears, etc.). Each problem was prefaced with the following instruction to establish the uniqueness of the relevant animal:

You are at an animal sanctuary. You have won a prize that allows you to get your picture taken alongside one of the animals at the sanctuary. This animal is going to be chosen at random by the staff at the sanctuary. However, you do not yet know which specific animal it will be. We would like you to indicate whether a set of assertions about this animal could all be true at the same time.

This framing occurred for each problem, and the participants answered the question, "Could all of these assertions be true at the same time?", responding either "Yes" or "No." An example of a problem with a basic conditional was accordingly:

If the animal is a dog then it has four legs.

The animal is not a dog.

The animal has four legs.

Could all of these assertions be true at the same time?

We re-iterate that the framing of all the problems concerned unique single entities.

3.1.3. *Design*

The participants' task was to judge whether all three assertions in a problem could be true at the same time. Approximately half of the participants (n = 46) were assigned at random to 16 problems based on an arbitrary assignment of contents to the four sorts of conditional. The remaining participants (n = 55) received a set of 16 problems based on a different arbitrary assignment of contents to the four sorts of conditional. The problems were presented in a different random order to each participant.

The experiment was administered in a web survey program (Qualtrics), and each problem was presented on a new onscreen page. The program recorded how long participants spent on each page, that is, the time they spent reading the assertions, inferring their responses, and then making their response. Upon completion of the experiment, participants provided some basic demographic information, and were thanked and debriefed. The program collected no other data.

3.2. Results and discussion

Table 3 presents the judgments and latencies for the sets of assertions with the conditionals of three sorts: basic, biconditional, and contingent. We collapsed the data across the two different assignments of contents to problems, and across the two sorts of contingent interpretations, as these variables had no reliable effects on performance. The results corroborated the model theory's three predictions. First, individuals judged that conditionals can be true in cases in which their *if*-clauses are false: 82 out of the 101 participants made more than 50% of such judgments in the predicted categories (see Table 3; Binomial test, p < .001). This result is not attributable to a response bias or to guessing, because the participants responded "No" correctly, also well above chance, to the assertions in the set, *If A then C*, *A*, *not-C* (see Table 3).

Second, for the basic and contingent conditionals, as Table 3 shows, individuals endorsed as collectively true the sets containing A and C more often and faster than the sets containing *not-A* and *not-C*, which in turn they endorsed more often and faster than the sets containing *not-A* and C. The trends were reliable both for the percentages of endorsement (Page's L = 1312.00, z = 7.04, p < .001) and for their latencies (Page's L = 1,043.00, z = 5.58, p < .001).

Third, assertions with a basic interpretation satisfied the predicted responses to a greater degree than assertions with a contingent interpretation. The crucial set of assertions is: *If A then C, not-A, C.* The basic conditionals yielded endorsements of this set 72% of the time, whereas the contingent conditionals yielded endorsements 51% of the

Table 3

The percentages of predicted judgments of whether all the assertions in a set could be true and their latencies in Experiment 2, depending on whether the conditional was basic, biconditional, or contingent, where the latter category included conditionals and their converses

The <i>If A then C</i> Assertion in the Set	Form of ond an Assertio	natical the Sec- d Third ns in the et	The Percentages of the Predicted Judgments of Whether All Assertions Can Be True	Latency of Predicted Judgments (s)
Basic	A	С	Yes: 100	8.17
	not-A	not-C	Yes: 81	9.83
	not-A	C	Yes: 72	11.35
	A	not-C	No: 92	7.71
Biconditional	A	C	Yes: 92	6.54
	not-A	not-C	Yes: 83	9.38
	not-A	C	No: 85	13.21
	A	not-C	No: 77	9.01
Contingent	A	C	Yes: 94	6.83
	not-A	not-C	Yes: 85	9.70
	not-A	C	Yes: 51	11.19
	A	not-C	No: 88	9.74

time, and the difference was reliable (Wilcoxon test, z = 4.38, p < .001). For contingent conditionals, the participants were evenly split between a basic and a biconditional interpretation: 33 made a basic interpretation (judging that the conditional was compatible with all possibilities except A and not-C), 32 made a biconditional interpretation (judging that the conditional was compatible with A and C, and not-A and not-C), and the remaining 36 participants responded with different interpretations across the problems. There was therefore no reliable trend toward one pattern of interpretation over any other, χ^2 (2) = .26, p = .88. Earlier studies have likewise reported that individuals often vacillate in this way (see Johnson-Laird et al., 1992).

Experiment 2 supported the model theory's predictions. Individuals reliably judged that it is possible for a conditional to be true even when its *if*-clause was false. The accuracy and latency results also supported the theory's subsidiary predictions concerning the order in which the models of a conditional should be fleshed out, and the different interpretations of the conditionals depending on their semantic contents. Theorists who support the defective truth table should consider the results with biconditionals. Standard biconditional interpretations of *If A then C* imply *If C then A* but also *If not-A then not-C* (see e.g., Evans, Handley, Neilens, & Over, 2008). Consistent with this interpretation for biconditionals, our participants evaluated *Not-A* and *Not-C* as capable of being true alongside *If A then C*. But, given the defective truth table, the conjunction of *If A then C* and *If C then A* is true only when *A* and *C* are both true, and otherwise has no truth value; and the conjunction of *If A then C* and *If not-A then not-C* can never be true. Hence, the defective truth table cannot account for these results.

4. Experiment 3

Experiment 3 aimed to corroborate the model theory using the collective truth task, but went beyond it to elicit judgments of collective probability as well. It used three sorts of conditional assertion: basic, contingent, or predictive. The predictive conditionals made predictions, such as: "If the legal drinking age in the United States is reduced, then there will be more traffic accidents." All three sorts should elicit a bias toward basic interpretations. The experiment elicited judgments of the collective truth of each set of assertions, as well as their collective probability, where a probability greater than zero implies that the assertions can all be true at the same time, no matter how rarely. A typical trial presented a conditional assertion, such as this predictive example:

If the legal drinking age in the United States is reduced, then there will be more traffic accidents.

It was followed on the same page by each of the four conjunctive contingencies, such as:

The legal drinking age in the United States is reduced and there are more traffic accidents.

Consider the following IF-THEN statement:

If the legal drinking age in the U.S.A. is reduced, then there will be more traffic accidents.

Now consider each of the four AND statements below. For each statement, please indicate whether it and the IF-THEN statement can BOTH BE TRUE. Alongside that judgment, please also indicate your estimate of the CHANCES that both are true. In the case that you judge that a pair of statements cannot both be true, then plainly the chances that they are both true must be 0.

You should use your general knowledge of the United States to inform your answers.

The IF-THEN statement is repeated here:

If the legal drinking age in the U.S.A. is reduced, then there will be more traffic accidents.

	Could both this statement and the IF-THEN statement above BOTH BE TRUE?		What are the chances that both this statement and the IF-THEN statement above are BOTH TRUE?
	Yes	No	Write a number from 0 - no chance at all to 100 - completely certain (write "0" if you indicated "No" to the left)
The legal drinking age in the U.S.A. is reduced and there are more traffic accidents.	0	0	
The legal drinking age in the U.S.A. is reduced and there are NOT more traffic accidents.	0	0	
The legal drinking age in the U.S.A. is NOT reduced and there are more traffic accidents.	0	0	
The legal drinking age in the U.S.A. is NOT reduced and there are NOT more traffic accidents.	0	0	

Fig. 1. A typical problem in Experiment 3, as presented on a single web-page.

The participants had to evaluate whether the conjunction and the conditional could both be true, and, subsequently, to rate the chances that they were both true (on a scale from 0 to 100). They carried out these two tasks for all four contingencies on the same webpage (see Fig. 1 above).

4.1. Method

4.1.1. Participants

One hundred and thirty-two participants in the United States began the experiment on Amazon.com's Mechanical Turk system in exchange for a small monetary payment. Twenty-six participants did not complete the experiment, and so their data were dropped from the analyses, yielding 106 participants (66 male, 40 female) in the experiment proper.

4.1.2. Design and materials

The participants served as their own controls and carried out four instances of each of three sorts of problem. Each problem consisted of an initial conditional assertion, If A then C, followed by conjunctions corresponding to the four possible contingencies in the partition: A and C, A and not-C, not-A and C, and not-A and not-C. There were three different sorts of problem depending on the nature of the conditional. The first sort contained basic conditionals, as in the previous experiment, in which the if-clause referred to a proper subset of the then-clause, for example, If the animal is a dog then it has four legs. The second sort contained contingent conditionals, as in the previous experiment, in which there was no prior relation between the referents of the if-clause and the thenclause, for example, If the animal has brown hair then it is a bear. The third sort contained conditionals that expressed a plausible prediction between two future events, for example, If the legal drinking age in the United States is reduced, then there will be more traffic accidents. Each of the three sorts of problem occurred with four different sorts of content, which are available from the first author upon request. The four different contingencies were presented in a random order for each problem, and the twelve problems were presented in a different random order to each participant.

4.1.3. Procedure

Fig. 1 presents a typical problem as it appeared to the participants on a web page. As the figure shows, the participants judged whether each of the four pairs of assertions could be true at the same time, and they also estimated the chances that both assertions were true, typing in a number ranging from 0 ("no chance at all") to 100 ("completely certain"). To enhance consistency between the two tasks, the instructions prompted participants to type "0" if they had responded, "No," to the previous question about collective truth.

Two practice problems preceded the main experiment. For these problems, participants made judgments only of collective truth, and they were asked a follow-up question about whether they had borne both statements in mind when making these judgments. The experiment was administered in a web survey program (Qualtrics), and each problem was presented on a new onscreen page. After the main experiment, participants provided some basic demographic information and were thanked and debriefed. The program collected no other data.

4.2. Results

Table 4 presents the percentages of predicted judgments of collective truth and collective probability for the three sorts of sets of assertions. The results corroborated the model theory's main prediction: Individuals judged that it is possible for a conditional to be true when its if-clause is false. Participants judged that the conditional could be true when both its clauses were true (A and C) on 96% of trials, and when its if-clause was false (not-A and not-C, 82%; not-A and C, 79%). Each of these three percentages was reliably greater than chance (50%; Wilcoxon tests, ps < .001 in all cases). These evaluations do not reflect a bias to respond, "Yes," because participants gave the correct "No" evaluation on 88% of the trials when the if-clause was true and the then-clause was false (A and not-C), and this tendency was also reliably greater than 50% (Wilcoxon test, p < .001). Overall, 92 out of 106 participants were more likely than chance (50%) to judge that the conditional could be true even when its if-clause was false (Binomial test, p < .001). As Table 4 shows, these same trends were observed consistently for each of the three sorts of conditional: basic, contingent, and predictive. There was only a small difference between judgments of the two sorts of false if-clause cases. Nonetheless, the predicted stochastic trend occurred: the participants endorsed as collectively true the pairs containing A and C more often than the pairs containing not-A and not-C, which in turn

Table 4

The percentages of predicted judgments of whether the two assertions in a set could both be true and their collective percentage probability in Experiment 3, depending on whether the conditional was basic, contingent, or predictive

The <i>If A then</i> C Assertion in the Set	Grammatical Form of the Second Assertion in the Set	The Percentages of the Predicted Judgments of Whether Both Assertions Can Be True	Mean Judged Probability That Both Assertions Could Be True
Basic	A and C	Yes: 98	85
	not-A and not-C	Yes: 82	60
	not-A and C	Yes: 86	59
	A and not-C	No: 86	14
Contingent	A and C	Yes: 96	81
	not-A and not-C	Yes: 83	63
	not-A and C	Yes: 80	51
	A and not-C	No: 74	23
Future Prediction	A and C	Yes: 95	75
	not-A and not-C	Yes: 80	56
	not-A and C	Yes: 72	42
	A and not-C	No: 73	23
Overall	A and C	Yes: 96	81
	not-A and not-C	Yes: 82	60
	not-A and C	Yes: 79	51
	A and not-C	No: 78	20

they endorsed more often than the pairs containing *not-A* and C (Page's L=1,348.00, z=5.22, p<.001). The participants' judgments of collective probability followed the same pattern. As Table 4 shows, they also corroborated the model theory's predicted stochastic trend (Page's L=1,411.00, z=9.55, p<.001). Once again, these percentages did not reflect a response bias because judgments of collective probability were much lower when the conditional was paired with A and not-C.

The four contingencies show a striking violation of the probability calculus: In principle, they should never sum to more than 100%, because the four contingencies correspond to those in the complete joint probability distribution. In fact, on average, participants' overall estimates summed to 211%, which was reliably much greater than 100% (Wilcoxon test, z = 8.74, p < .001). This sort of subadditivity (in which the probability of the parts is greater than that of the sum) often occurs, prompting Tversky and Koehler (1994) to develop "support theory" to explain it, but it is seldom as large as the present magnitudes (Rottenstreich & Tversky, 1997). Furthermore, our instruction to participants to respond "0" if their judgments of collective truth were "No" should bias against this result. A similar massive violation of the probability calculus occurred in a study in which the participants evaluated inferences of conjunctions from disjunctive premises: their estimates of the probabilities of each conjunction in the disjunction's partition summed to a mean of 191% (see Experiment 2, Hinterecker, Knauff, & Johnson-Laird, 2016). Elsewhere, the model theory explains the processes underlying numerical estimates of the probabilities of unique events such as those in the present experiment, and both the theory and its implementation in the mReasoner computer program predict subadditivity (Khemlani, Lotstein, & Johnson-Laird, 2015).

4.3. Discussion

Like the previous experiment, the present results corroborated the model theory and ran counter to the defective truth table. Participants evaluated conditionals as true even when their *if*-clauses were false, and their estimates of the probabilities corroborated these evaluations. These evaluations occurred for conditionals that expressed relations between the existing properties of a single entity, such as:

If this animal is a fish then it has blue fins;

as well as for everyday conditionals making predictions about future events, such as:

If the U.S. school day is lengthened, then U.S. school children's standardized test scores will rise.

Likewise, as in the previous experiment, the tendency to evaluate a conditional as "True" reflected the order in which mental models should be fleshed out as fully explicit models.

By contrast with Experiment 2, the contingent conditionals yielded a greater percentage of conditional interpretations. This difference may reflect the contexts of the two experiments. In Experiment 2, the presence of explicit biconditionals in some problems may have inflated the percentage of biconditional interpretations of the contingent

conditionals in comparison with the present experiment, in which there were no explicit biconditionals.

A recent burgeoning of probabilistic approaches to reasoning has proposed that the probability of a conditional, If A then C, is equal to the conditional probability $P(C \mid A)$ – a claim made by many psychologists (e.g., Evans & Over, 2004; Fugard et al., 2011; Oaksford & Chater, 2007). However, the degree of subadditivity observed in the present experiment implies that naïve individuals' numerical estimates of conditional probabilities cannot be related in a sensible way to their estimates of the probabilities of their component parts (see also Girotto & Johnson-Laird, 2004; Khemlani et al., 2015; and Hinterecker et al., 2016; for further corroboration). Hence, studies in which participants' estimates of the four contingencies are constrained by the explicit instruction that these estimates should sum to 100% are likely to yield ecologically invalid results (cf. Over, Hadjichristidis, Evans, Handley, & Sloman, 2007). We therefore take the view expressed in Sanborn and Chater (2016) that brains need not calculate numerical probabilities at all, and are poorly adapted to do so (see also Khemlani et al., 2015; Oaksford & Hall, 2016).

5. Experiment 4

Our final experiment contrasted the model theory and the defective truth table in a new way based on the relation between presuppositions and suppositions. Consider the following pair of assertions:

The man's wife is older than he is.

The man does not have a wife.

The first assertion presupposes that the man has a wife, and so the second assertion conflicts with this presupposition. If the second assertion is true, then the first assertion cannot be true — it is either false (Russell, 1905) or has no truth value (Strawson, 1950). In either case, the first assertion can be true only if the second assertion is false; that is, the man has a wife (Beaver & Geurts, 2013). Hence, the assertions cannot be true at the same time, and individuals should make this judgment in the collective truth task. Now, consider this pair of assertions:

If the man has a wife, then she is older than he is.

The man does not have a wife.

According to the suppositional theory of conditionals — one of the leading theories defending the defective truth table — the *if*-clause makes a supposition, and if that supposition is false, the conditional has no truth value (see e.g., Evans & Over, 2004). Hence, the falsity of the second assertion (and, consequently, the truth of the *if*-clause of the conditional) is a necessary condition for the conditional to be true or false (see Table 1). Hence, on this account, participants should reject the possibility that both

assertions can be true. In other words, the suppositional function of the *if*-clause of a conditional is thought to be analogous to its presupposition. In contrast, according to the model theory, participants should accept that both assertions can be true, and so there is no direct analogy with such a presupposition. The conditional can be true even in the case that the man has no wife: Its *then*-clause applies only in the case that he does have a wife. Our final experiment therefore contrasted the role of suppositions in the *if*-clauses of conditionals with the role of suppositions in non-conditional assertions, as in the first pair of assertions above. The model theory predicts endorsement of collective truth for the conditional pairs, but denial of collective truth for the non-conditional pairs.

5.1. Method

5.1.1. Participants

One hundred and ten participants in the United States began the experiment on Amazon.com's Mechanical Turk system in exchange for a small monetary payment. Ten participants did not complete the experiment, and so their partial data were removed from the analyses, yielding 100 participants (63 male, 37 female) in the experiment proper.

5.1.2. Design and materials

Participants were randomly assigned to one of two sets of problems: conditionals and non-conditionals. In both sets, each problem consisted of two assertions, followed by a question about whether both assertions could be true at the same time. If participants responded that the two assertions could not both be true at the same, they received a follow-up question asking them to explain why. There were 16 problems in each set, consisting of eight target problems and eight fillers. As Table 5 shows, the contents of the problems were matched across the conditional and non-conditional sets. Because the eight target problems were all likely to yield the same answer, the filler problems were included to balance responses. Their contents were expected to yield answers opposite to those that the model theory predicts for the target problems. Hence, the fillers for the conditional problems were designed to yield "No" answers. A typical filler was: *If the woman eats meat, then her favorite meat is beef. The woman's favorite meat is chicken.* The first assertion has the force of a biconditional, because in case the woman's favorite meat is beef, then she must also eat meat. The conditional accordingly refers to two possibilities:

Woman eats meat and her favorite meat is beef

Not (woman eats meat)

The second assertion also presupposes that the woman eats meat and asserts that her favorite meat is chicken, and so it is inconsistent with both possibilities for the conditional. Hence, participants should respond that both assertions cannot be true at the same time.

Table 5 The contents of the conditional and non-conditional problems presented to two separate groups in Experiment 4, the percentages of judgments that the pairs could both be true, and the results of Mann–Whitney U-tests comparing the judgments

Problem Number	Contents	Percentage of Judgments That Both Assertions Could Be True	Conditional Versus Non-Conditional Problems
1	Conditional: If the man has a wife, then she is older than he is. The man does not have a wife. Non-Conditional: The man's wife is older than he is. The man does not have a wife.	86 8	z = 7.78, p < .001
2	Conditional: If the boy has a dog, then it is a spaniel. The boy does not have a dog. Non-Conditional: The boy's dog is a spaniel. The boy does not have a dog.	82 6	z = 7.62, p < .001
3	Conditional: If the man drinks alcohol, then his usual alcoholic drink is beer. The man does not drink alcohol Non-Conditional: The man's usual alcoholic drink is beer. The man does not drink alcohol.	70 16	z = 5.43, p < .001
4	Conditional: If the apartment has two floors, then it's second floor is a roof deck. The apartment does not have two floors. Non-Conditional: The apartment's second floor is a roof deck. The apartment does not have two floors.	76 22	z = 5.37, p < .001
5	Conditional: If the plane has a first-class cabin, then it is full. The plane does not have a first-class cabin. Non-Conditional: The plane's first class cabin is full. The plane does not have a first class cabin.	80	z = 7.22, p < .001
6	Conditional: If the game is on today, then it is scheduled for this afternoon. The game is not on today. Non-Conditional: The game is scheduled for this afternoon. The game is not on today.	78 12	z = 6.60, p < .001
7	Conditional: If the patient has an enlarged heart, then the cause is genetic. The patient does not have an enlarged heart. Non-Conditional: The patient's enlarged heart has a genetic cause. The patient does not have an enlarged heart.	82	z = 7.84, p < .001
8	Conditional: If the band has a manager, then he is the trumpet player. The band does not have a manager. Non-Conditional: The band's manager is the trumpet player. The band does not have a manager.	86 10	z = 7.57, p < .001

In contrast, the fillers for the non-conditional problems were designed to yield "Yes" answers (in order to balance the expected "No" responses in that condition), e.g.: *The woman's favorite meat is beef. The woman's favorite vegetable is cabbage.* Table 5 above shows the contents for each experimental problem.

5.1.3. Procedure

The experiment was administered in a web survey program (Qualtrics), and each problem was presented on a new onscreen page. In both groups, the two assertions were following by the question:

Can these two statements both be true at the same time?

There were two response options: Yes, they can both be true; and No, they cannot both be true. For trials in which participants responded, "No," they received a follow-up question which asked them to select the reason why from four options: (a) because the second statement makes the first one false, (b) because the second statement makes the first one neither true nor false, (c) for some other reason, which the participants then typed, and (d) "Not sure." After the experiment, participants provided some basic demographic information, and were thanked and debriefed. The program collected no other data.

5.2. Results and discussion

Table 5 presents the contents of the conditional and non-conditional problems in the two groups in Experiment 4, the percentages of judgments that the pairs could both be true, and the results of Mann–Whitney U-tests comparing the judgments. Overall, the participants judged that the conditionals could be true when their *if*-clauses were false on 80% of trials (reliably greater than chance, Wilcoxon test, p < .001), whereas they judged that the non-conditionals could be true when their presuppositions were false on only 11% of trials (reliably less than chance, Wilcoxon test, p < .001). This difference between the two groups was highly reliable (Mann–Whitney U test, z = 7.77, p < .001), and, as Table 5 shows, it was also highly reliable for each of the eight matched content pairs. When the participants responded that the non-conditional problems could not both be true together, their reasons were that the second assertion made the first assertion "False" (91%) rather than "Neither true nor false" (8%).

Once more, individuals judged that conditionals can be true when they have false *if*-clauses, but they judged that analogous pairs of assertions — in which one assertion denied a presupposition of the other — could not both be true. These results therefore refute the analogy between conditionals and presuppositions implied by the suppositional account of the meaning of conditionals (e.g., Evans & Over, 2004). The result generalizes those of the previous studies to new sorts of conditionals and to a new condition in which information was provided only about the falsity of the conditional's *if*-clause, with no information provided about the conditional's *then*-clause. In sum, the results show that individuals can interpret a conditional without supposing that its *if*-clause is true (or adding it to their "stock of knowledge," as in Ramsey, 1990/1926, famous footnote). They

judged that a conditional could be true even when its *if*-clause was false. This result therefore refutes a foundational claim of the suppositional theory of conditionals and casts further doubt on the defective truth table.

6. General discussion

Studies of the truth of conditionals recently seemed to be at an impasse. On the one hand, tasks that assess the circumstances in which conditionals are true — the truth-table task and its cognates — yield results suggesting a defective truth table in which conditionals are neither true nor false when their if-clauses are false (see Table 1). On the other hand, tasks that assess the possibilities to which conditionals refer yield results that include possibilities in which the if-clause is false. The two tasks ought to yield compatible results, because if a situation is possible according to a conditional, then it should also be one in which the conditional is true. One view is to prioritize one or other of the tasks as revealing the true meaning of conditionals. Indeed, our results stem, in part, from a critique of the standard truth-table task, and from an effort to improve it, as well as to devise a new task that overcomes its problems. Another view, as we mentioned in the Introduction, is that the two tasks tap into different abilities, and so their results are not discrepant (Barrouillet et al., 2008). A still further view is that compound assertions, including conditionals, refer to various possibilities, and so it may be odd to ask individuals to assess their truth from just a single piece of evidence (Schroyens, 2010a). In fact, the model theory leads to a new synthesis about the meaning of compounds, which we outline below. First, though, we consider the results of our experiments.

Granted that the conventional truth-table task may not be felicitous, we devised two tasks to determine the circumstances in which conditionals can be true. In the revised truth-table task, investigated in Experiment 1, the participants learn an outcome of a choice, such as:

In the end, John fired Annie.

They use this evidence to evaluate a conditional prediction:

John will fire Charlie, if not Annie.

as true, false, or irrelevant. The use of a negative *if*-clause was intended to trigger the corresponding affirmative contingency too (as suggested by the results of Experiment 3 in Johnson-Laird et al., 1992). And the use of an ellipsis (a partial clause) in the *if*-clause (as a result of the *q if not p* formulation) was intended to minimize any tendency to take the *if*-clause for granted and merely to evaluate the *then*-clause. Most of the participants' evaluations in cases in which the *if*-clause was false (as in the example above) were that the conditional was true, and there were few evaluations of "Irrelevant" (see Experiment 1) — in stark contrast to the results of experiments with the standard truth-table task. A major result of Schroyens's (2010b) meta-analysis of the truth-table task was that, contrary to claims that still occur in the literature (e.g., Milne, 2012), the defective truth table is not its invariable result. The present results corroborate this point.

A more appropriate method to determine the truth conditions of specific conditionals is to present a conditional in a set of assertions and to ask participants: *Could* all of these assertions be true at the same time? We used this task in Experiments 2–4. It does not seek an evaluation of definite truth, but merely its possibility. The model theory postulates that individuals search for a model in which all the assertions hold. It should therefore be easy to respond, "Yes," when the two categorical assertions render both clauses in the conditional true, because this conjunction is equivalent to the one explicit mental model of the conditional. With other categoricals, however, individuals have to flesh out their mental models into fully explicit models. The theory postulates that they do so in the order: *not-A and not-C* and then *not-A and C*. It takes work to carry out this process, and so the theory predicts a corresponding declining trend in "Yes" evaluations. Our experiments corroborated this trend in evaluations (Experiments 2 and 3), their latencies (Experiment 2), and their probabilities (Experiment 3).

Skeptics might argue that the collective truth task is not a natural test of meaning. Its use in other studies, however, seems uncontroversial (see e.g., Johnson-Laird et al., 2000). Other skeptics might argue that the collective truth task is merely a variant of the listing of possibilities (as in, e.g., Lecas & Barrouillet, 1999; Quelhas et al., 2010), and so it is no wonder that the two tasks yield similar results. However, this criticism overlooks a fundamental distinction between tasks that call merely for listing possibilities and those that call for assessments of truth. As Barrouillet et al. (2008) have suggested, these tasks represent "two different kinds of reasoning" (p. 761); "the former kind [reasoning about possibilities] would be psychologically basic, whereas the latter [reasoning about truth] is more complex and difficult, involving some form of metalogic, a meta-ability that requires a grasp of the relations between assertions and the world through the predicates true and false" (Johnson-Laird & Byrne, 2002; Moshman, 1990). The critical feature of the collective truth task is that individuals do not have to show that a conditional is true, but merely that it could be true. Thus, they must consider the relations between assertions in the entire set in order to arrive at the right judgment for the right reasons. Hence, skeptics owe readers an account of how people make judgments about truth values from listing possibilities. Consider a problem of this sort:

If the structure of the house is weak then it will collapse in an earthquake.

The structure of the house is weak.

The house will collapse in an earthquake.

Could all these assertions be true at the same time?

The judgment goes beyond the listing the possibilities consistent with the conditional assertion. It calls for individuals to grasp the sets of possibilities for each assertion, and to determine whether there is at least one possibility common to all of them. If they find such a model, they respond, "Yes, the assertions could all be true." Otherwise, they respond, "No, the assertions could not all be true." In short, it suffices to show that there is a possibility common to all the assertions in order to infer that a set of assertions could all be true.

Some philosophers have defended the view that everyday conditionals express the material implication of logic (e.g., Grice, 1989, p. 58 et seq.). It has the truth table shown in Table 1. An assertion that A materially implies C is true for those cases in which not-A holds, and it is also true for those cases in which C holds. Suppose, for instance, you know that a particular proposition, C, is true, should you judge that a conditional, If A then C, is true solely in light of this evidence? Material implication entails that you should, but its unacceptability is widely recognized: The inference is known as a "paradox" of material implication, and reasoners tend to reject it (Orenes & Johnson-Laird, 2012; Schroyens, 2010a). Many critics of the theory of mental models have argued that it too is committed to material implication (see e.g., van Wijnbergen-Huitink, Elqayam, & Over, 2015). There is indeed an analogy between a truth table for material implication and the model theory's possibilities for If A then C. As Table 1 shows, a material implication is true in just those cases that are possible. But analogy is not identity. Possibilities are not truth values. Johnson-Laird and Byrne (2002) accordingly distinguished between everyday conditionals and material implications (p. 655), and they rejected the notion that conditionals have any sort of truth-functional meaning (p. 673). Likewise, the fact that cases in which the if-clause is false are possible for both true and false conditionals is also contrary to material implication (Quelhas et al., 2017).

Overall, for both the truth task and the collective truth task, there was a dearth of evaluations corresponding to the defective truth table: 5% of trials in Experiment 1, 9% of trials in Experiment 2, and 5% of trials in Experiment 3. The evaluations in Experiment 4 concerned only cases of false *if*-clauses, and again most participants judged conditionals to be true in this case, whereas in a parallel case of a failure of a presupposition, they judged a non-conditional assertion to be false. The suppositional account of the meaning of conditionals (e.g., Evans & Over, 2004) implies a similarity in the interpretative process for these two sorts of assertion. Yet our results revealed a striking divergence: Participants judged that conditionals could be true when their suppositions (*if*-clauses) were false, but they judged that non-conditional assertions could not be true when their presuppositions were false. Overall, the results of the four experiments refuted the defective truth table and corroborated the possibilities that reasoners list given the truth of conditionals. Conditionals can be true when the situations to which they refer are possible.

In the Introduction, we described three interpretations of the defective (or de Finetti) truth table in Table 1. Cases in which its *if*-clause is false can mean that the conditional is void and has no truth value, that the conditional's truth value has yet to be determined, or that it has some value other than truth or falsity. In the latter case, say for example that the conditional expresses a conditional probability, it is unclear in what circumstances the conditional is true, or even if such circumstances exist in principle (see Adams, 1998).

The first of these interpretations (void) has been expressed most frequently by a variety of theorists in the new paradigm who favor a de Finetti semantics for the conditional (e.g., Evans, 2003; Evans & Over, 2004, p. 153; Fugard et al., 2011; Oaksford & Chater, 2007; p. 109; Politzer et al., 2010). Our results run counter to this interpretation, which implies that the conditional cannot be true when its if-clause is false. The sole way a

conditional can be true on this account is when its *if*-clause and *then*-clause are both true. Accordingly, the participants in our studies should have judged that only in such a case could all the assertions be true at the same time, and they should have judged that each of the other sets, including those in which the *if*-clause did not hold, could not all be true at the same time. They did not make these evaluations.

Our evidence similarly poses a problem for the third (distinct truth value) interpretation, for which it is unclear how a conditional could be true in conjunction with its false antecedent. For the second interpretation (truth value yet to be determined), matters are perhaps less clear. Readers may think that this view could accommodate the present results, particularly if "undetermined" is taken to mean "not yet known" (epistemic uncertainty) rather than "not yet existent" (metaphysical uncertainty). However, proponents of such a view lack an account of what else would need to be established, or to happen, in order to determine the conditional's truth value when its antecedent is false. Otherwise, this view seems to collapse into the first (void) interpretation, and to thereby similarly run afoul of our evidence.

We are now, at last, in a position to explain how the model theory fits together possibilities, truth values, and probabilities for conditionals that refer to specific outcomes or entities. The theory bases the meaning of conditionals and other compounds on possibilities. As Table 1 above shows, a true conditional, such as:

If Viv has shingles then she is in pain

entails the following conjunction of three possibilities and an impossibility for Viv's predicament:

shingles	pain	(Possible)
shingles	no-pain	(Not possible)
not-shingles	pain	(Possible)
not-shingles	no-pain	(Possible)

That is, given that the conditional above is true, the conjunction of these three possibilities and one impossibility should hold. The assertion of this conditional should not be taken as expressing the idea that having shingles without pain is impossible in some absolute sense. Rather, what these semantics indicate is simply that the truth of the conditional in this specific instance is incompatible with Viv's having shingles without pain. But, as we mentioned in the Introduction, Quelhas et al. (2017) showed that when a conditional is false, the cases in which the *if*-clause is false are also possible. Thus, the falsity of the conditional above — which refers to a specific event — entails the following conjunction:

shingles	pain	(Not possible)
shingles	no-pain	(Possible)
not-shingles	pain	(Possible)
not-shingles	no-pain	(Possible)

The third case is the possibility in which Viv doesn't have shingles and is in pain from some other cause, and the fourth case is the possibility in which Viv doesn't have shingles and is not in pain. As these two sets of possibilities show, the cases in which the *if*-clause of the conditional is false are possible whether the conditional is true (in the first set) or false (in the second set).

Thus, for conditionals that refer to specific events, outcomes or individuals, which have been the primary focus of recent controversies, these sets of possibilities reconcile the results of the task of listing possibilities with the results of the truth-table task. Individuals list the three possibilities for true conditionals shown above and in Table 1. They judge two of them — those in which the *if*-clause of the conditional is false — as irrelevant to the truth of the conditional in the truth-table task. They are possible whether the conditional is true or false, and so they cannot fix its truth value. Studies of the truth-table task have found that, for false *if*-clauses, intelligent individuals are more likely to make judgments of "irrelevance" than less intelligent individuals (Evans, Handley, Neilens, & Over, 2007). These investigators took this result to count against the model theory (but cf. Sevenants, Dieussaert, & Schaeken, 2013). In fact, these cases are also possible for false conditionals. The present account therefore predicts that intelligent individuals should judge them as irrelevant.

The set of possibilities for true conditionals makes sense of the collective judgments of truth values in our experiments. The following assertions:

If Viv has shingles then she is in pain.

Viv does not have shingles.

Viv is not in pain.

can all be true at the same time, because the two categorical assertions are both possible given the truth of the conditional. Thus, in what circumstances is a conditional about a particular individual or entity true? The two sets of possibilities imply that when the conditional, *If Viv has shingles then she is in pain*, is true, it is possible that Viv has shingles and is in pain, and it is impossible that she has shingles and is not in pain. The observation that she has shingles and is in pain goes beyond a possibility to a fact, which also eliminates the impossible case. Hence, the conditional is true. The observation that Viv does not have shingles is consistent with the truth or falsity of the conditional. But, as our experiments showed, people do judge that the conditional could be true in case Viv does not have shingles. And it would be true given that the corresponding counterfactual is true:

If Viv had had shingles then she would have been in pain.

It refers to a counterfactual possibility — one that was once a real possibility, but that didn't happen (Johnson-Laird & Byrne, 2002) — and so it is true. These truth conditions are in striking contrast to those of orthodox logic in which it suffices only to eliminate as impossible cases of people with shingles who do not experience pain, and so the conditional can be true in case no-one has shingles (see, e.g., Jeffrey, 1981, p. 116).

The model theory yields an account of specific conditionals that differs from the probabilistic conditional, which implies that the conditional's meaning is a conditional probability. It differs from material implication and from the defective truth table: Both of them imply that if a conditional is false its *if*-clause is true (see Table 1). In that case, the following short proof would be valid.

It is not the case that if God exists then atheism is correct.

Therefore, God exists.

In sum, conditionals are not probability conditionals (pace Adams, 1998; Pfeifer & Kleiter, 2009), not defective truth functions (pace, e.g., Evans & Over, 2004) and not material implications (pace Grice, 1989).

Because false *if*-clauses hold whether a conditional is true or false, they are irrelevant to its probability, which depends solely on the proportion of chances in which A and C hold out of the overall chances of A. Hence, in theory, the probability of a conditional equals the well-known Equation that probabilists espouse, namely, p(If A then C) = p(C given A). Does it follow that the model theory has undergone a conversion into the probabilistic conditional? Not at all. We have already rejected its account of the meaning of conditionals. In Experiment 3, our participants made estimates of the probabilities of each of the four contingencies in the partition of conditionals. Their estimates ought to have summed to 100%. In fact, they typically summed to over 200%. Such a massive violation of the probability calculus runs counter to any account, such as the new paradigm, that places numerical probabilities at the center of reasoning. They are, however, in accordance with claims due to other proponents of the turn to probabilities: "Bayesian brains need not represent or calculate probabilities at all and are, indeed poorly adapted to do so. . . . with finite samples, [the brain] systematically generates classic probabilistic reasoning errors" (Sanborn & Chater, 2016, p. 883).

In conclusion, our findings imply a new answer to the question: what is the correct meaning of basic conditionals that refer to specific outcomes or entities — the de Finetti truth table or the truth table for material implication? The answer is: neither. The correct meaning emerges from the listing of possibilities, the revised truth-table task, and the collective truth task. Their results refute the de Finetti truth table, because a basic conditional can be true when its *if*-clause is false. But this finding does not imply that basic conditionals about particular individuals are material implications. Our results suggest instead that they are true when their mental model corresponds to the facts or to a counterfactual possibility.

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Note

1. Unlike the other four problems, this last problem was not framed explicitly as an inclusive disjunction. However, participants appeared no less likely to treat it as an inclusive disjunction, as revealed by their judgments of whether the conditional prediction was true when its antecedent was false and its consequent true (air strike and ground invasion; 55% for this problem vs. 55.9% for the other problems, Wilcoxon test, z = 0.33, p = .75). We, therefore, averaged responses for all five problems together.

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