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The Analytic Truth and Falsity of Disjunctions

Ana Cristina Quelhas,[a] Célia Rasga,[a] P.N. Johnson-Laird[b,c]*

[a]*Applied Psychology Research Center, Capabilities & Inclusion, ISPA-Instituto Universitário*
[b]*Department of Psychology, Princeton University*
[c]*Department of Psychology, New York University*

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Abstract

Disjunctive inferences are difficult. According to the theory of mental models, it is because of the alternative possibilities to which disjunctions refer. Three experiments corroborated further predictions of the mental model theory. Participants judged that disjunctions, such as *Either this year is a leap year or it is a common year* are true. Given a disjunction such as *Either A or B*, they tended to evaluate the four cases in its 'partition': *A and B*, *A and not-B*, *not-A and B*, *not-A and not-B*, as 'possible' or 'impossible' in ways that bore out the difference between inclusive disjunctions ('or both') and exclusive disjunctions ('but not both'). Knowledge usually concerns what is true, and so when participants judge that a disjunction is false, or contingent, and evaluate the cases in its partition, they depend on inferences that yield predictable errors. They tended to judge that disjunctions, such as follows: *Either the food is cold or else it is tepid, but not both*, are true, though in fact they could be false. They tended to infer 'mirror-image' evaluations that yield the same possibilities for false disjunctions as those for true disjunctions. The article considers the implications of these results for alternative theories based on classical logic or on the probability calculus.

Keywords: Analyticity; Cognitive illusions; Disjunctions; Mental models; Modulation; Possibilities

1. Introduction

Disjunctions can be difficult to understand (e.g., Johnson-Laird, 1983, p. 424), to make inferences from (e.g., Johnson-Laird, Byrne, & Schaeken, 1992), and to base decisions on (e.g., Tversky & Shafir, 1992). Consider this disjunctive assertion:

Either the food is cold or else it is tepid, but not both.

Correspondence should be sent to Ana Cristina Quelhas, ISPA-Instituto Universitário, Applied Psychology Research Center, Capabilities & Inclusion, Rua Jardim do Tabaco, 34, 1149-041 Lisbon, Portugal. E-mail: cquelhas@ispa.pt

*P.N. Johnson-Laird is a visiting scholar in New York University

Is it true, is it false, or is it impossible to determine which it is? The question calls for some thought, and people often answer that the disjunction is true. For reasons to be explained presently, they are wrong. Likewise, to formulate a conclusion that follows from a pair of disjunctive premises is very difficult (Bauer & Johnson-Laird, 1993), to formulate a conclusion that follows from a single disjunctive premise is quite difficult (e.g., Johnson-Laird, Byrne, & Schaeken, 1992), and to formulate a conclusion that follows from a conjunction is less difficult (García-Madruga, Moreno-Rios, Carriedo, Gutiérrez, & Johnson-Laird, 2001). From the standpoint of theories of reasoning based on classical logic (e.g., Braine & O'Brien, 1998), the difficulty is mysterious, because formal rules of inference for disjunction and conjunction seem similar (cf. Rips, 1994, pp. 365–369). Likewise, from the standpoint of theories of reasoning based on probabilistic logic (e.g., Cruz, Over, & Oaksford, 2017; Oaksford & Chater, 2007; Pfeifer, 2013), estimates of the probability of a disjunction and the probability of a conjunction are of comparable difficulty (Khemlani, Lotstein, & Johnson-Laird, 2015). So theories based on classical logic or the probability calculus do not solve the mystery.

In contrast, the theory of mental models—the 'mental model' theory, for short (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015)—explains the difficulty. It postulates that compound assertions refer to sets of exhaustive possibilities, which hold in default of knowledge to the contrary. Exclusive disjunctions of the sort: *Either A or else B, but not both*, call for models of two possibilities, and inclusive disjunctions of the sort: *A or B, or both*, call for models of three possibilities. These numbers of possibilities suffice to cause difficulties in comparison with conjunctions of the sort, *A and B*, which call for only one model. One consequence is that inferences from exclusive disjunctions are easier than those from inclusive disjunctions (Johnson-Laird et al., 1992). But the mental model theory makes further predictions. It predicts that the meanings of clauses in disjunctive assertions and general knowledge can modulate interpretation. Some disjunctions should therefore be judged to be true (or false), and some should be judged to be contingent; that is, their truth or falsity is impossible to determine without evidence. The theory also predicts that different cases should be judged as possible depending on whether a disjunction is inclusive or exclusive, and on whether it is true or false.

These studies used two tasks to test these predictions. The first task called for participants to judge whether a disjunction is true, false, or contingent. A previous study had used the task to examine conditional assertions, such as:

1. If Mary has the flu, then she is ill.

which most participants judged to be true. And they judged its counterpart:

2. If Mary has the flu, then she is healthy.

to be false (Quelhas, Rasga, & Johnson-Laird, 2017). But proponents of the probabilistic approach to reasoning argue that conditionals are exceptional: They differ in meaning from their counterparts in classical logic, whereas other compounds such as disjunctions do not (e.g., Evans & Over, 2004; Oaksford & Chater, 2007). In particular, Adams (1998) argued that conditionals are not really true or false—they have only 'ersatz' truth values—and that their real meanings are conditional probabilities. Others have defended this view too (e.g., Pfeifer, 2013). So we used the task to test whether the mental model theory's predictions applied to disjunctions, which, as all parties agree, have genuine truth values.

The second of the two tasks in the present studies called for the participants to evaluate each of a set of cases as 'possible' or 'impossible' given a disjunction. For example, given the disjunction, 'Either the book is brand new or it is used,' the participants had to evaluate each of the four cases in its *partition*:

- The book is brand new and it is used.
- The book is brand new and it is not used.
- The book is not brand new and it is used.
- The book is not brand new and it is not used.

In general, the partition of any compound assertion of one clause, *A,* with another clause, *B*, consists of the exhaustive set of conjunctions of one clause or its negation with the other clause or its negation: *A and B*, *A and not-B*, *not-A and B*, and *not-A and not-B*. These are merely assertions that are true or false depending on the truth value of the compound. The task, however, called for each of these conjunctions to be evaluated as possible or impossible for disjunctions that participants judge to be true, and on separate trials for disjunctions that they judge to be false.

The overall goal was to show that the mental model theory solves the mystery of disjunctions. The research had three immediate aims. The first aim was to test whether, as the theory predicts, certain disjunctions are true, others are false, and still others are contingent—their truth value is impossible to determine in the absence of evidence. The theory predicts that individuals should judge certain disjunctions to be true even though they could be false; that is, there are illusions of 'truth'. The second aim was to test the theory's account of the different possibilities to which distinct sorts of disjunction refer, and its prediction of a 'mirror-image' error in which the participants' possibilities for false disjunctions turn out to be the same as those they assign to true disjunctions. The third aim was to test the prediction that judgments of truth value, and evaluations of possibilities, should be easier for true disjunctions than for false disjunctions, because individuals have to infer that disjunctions are false and to infer their possibilities. In what follows, we outline the mental model theory in order to derive these predictions. We then report three experiments designed to test them, but which also yielded an unexpected result. And, finally, we discuss the implications of the results for theories of disjunction.

1.1. The mental model theory

The mental model theory postulates that assertions refer to possibilities, which are represented in models, and that reasoners use these models to make inferences. Models have a structure that insofar as possible is the same as the structure of what they represent—they are 'iconic', and so an inclusive disjunction, such as:

There is a triangle or a square on the board, or both.

refers to three exhaustive possibilities that are represented in three models of what could be on the board. The following diagram of the models presents these possibilities on separate rows:

△

□

△ □

One case is impossible given the disjunction above:

¬ △ ¬ □

where the symbol, '¬', denotes negation, so the disjunction also means that it is impossible that there is neither a triangle nor a square on the board. Each of the possibilities holds in default of knowledge to the contrary, and together the four cases are in a conjunction, because possibilities can be conjoined even when their constituent propositions are incompatible with one another, e.g., it possible that it is raining and it is possible that it is not raining.

The models above imply the following four inferences:

Therefore, it is possible that there is a triangle on the board.
Therefore, it is possible that there is a square on the board.
Therefore, it is possible that there is both a triangle and a square on the board.
Therefore, it is impossible that there is neither a triangle nor a square on the board.

Individuals tend to make all four of these inferences (Hinterecker, Knauff, & Johnson-Laird, 2016). They seem so unremarkable that some critics have argued that they are valid, where validity guarantees that if the premises of an inference are true then so too is its conclusion (Jeffrey, 1981, p. 1). In fact, none of the inferences is valid in any standard logic dealing with possibilities and disjunctions, that is, a modal sentential logic (see Girle, 2009). To see why, consider the situation in which it is impossible that there is a triangle on the board but there is a square on the board: The disjunctive premise above is true (i.e., *There is a triangle or a square on the board, or both*), but the first conclusion is false (i.e., *It is possible that there is a triangle on the board*). So the inference is invalid. Analogous situations show that the other conclusions about possibilities can also be false. Probabilistic logic replaces validity with the concept of probabilistic validity (*p*-validity). It demands that, given a consistent assignment of probabilities to their clauses, inferences from a single premise should never yield a conclusion less probable than the premises. Yet, from the participants' own estimates of the probabilities, most of their inferences (67% of them) violated this principle (Hinterecker et al., 2016).

The mental model theory postulates that there are two systems for reasoning—an idea that the late Peter Wason first proposed. The two systems underlie an algorithmic account of performance in his well-known selection task (Johnson-Laird & Wason, 1970). Nearly 50 years later, a meta-analysis of over 200 experiments investigating the selection task showed that this algorithm fit the data better than any other theory (Ragni, Kola, & Johnson-Laird, 2018). Many dual-process theories of reasoning now exist (see Evans, 2008)—of which the most famous is due to Kahneman (2011). None of these alternatives has an algorithmic implementation.

In the mental model theory, system (a), the intuitive system, uses mental models such as those above, but for simple tasks, such as evaluating the possibilities to which true assertions refer, individuals can use system (b), the deliberative system, which has more computational power; that is, it can use working memory to compare alternative models (see, e.g., Khemlani & Johnson-Laird, 2017). In system (b), models are fully explicit in that they represent the truth or falsity of each clause in the premises in each model of a possibility. For example, our earlier disjunction, 'There is a triangle or a square on the board, or both,' has the following conjunction of *fully explicit* models of the three exhaustive possibilities, which each hold in default of knowledge to the contrary, and where negation is used to represent falsity:

$$\begin{array}{ll} \triangle & \neg\square \\ \neg\triangle & \square \\ \triangle & \square \end{array}$$

The impossible case remains as follows:

$$\neg\triangle \quad \neg\square$$

When a disjunction is false, its negation is true, and so the falsity of our exemplary disjunction means that only one state of affairs is possible: There is neither a triangle nor a square. A critical consequence of the theory for our studies is that performance with true disjunctions should be easier than performance with false or contingent disjunctions. The reason is that individuals have immediate access only to knowedge of what has to hold for a compound assertion to be true, and so they have to infer this information for a false compound assertion.

In sum, the mental model theory postulates that compound assertions, such as disjunctions, refer to conjunctions of possibilities and impossibilities, which hold in default of knowledge to the contrary. So, everyday reasoning is defeasible: people draw conclusions that they are happy to withdraw if hard facts contradict them—the mental model theory is therefore 'nonmonotonic'. In contrast, classical logic is 'monotonic,' because each new premise can lead only to further conclusions, and never to the withdrawal of a conclusion. The mental model theory's algorithm is implemented in a computer program, mSentential, which is available from http://mentalmodels.princeton.edu. We now describe what is needed to understand the theory's predictions for the present studies.

1.2. The possibilities for different sorts of disjunction

Three main sorts of disjunction occur in daily life, and we give examples of each in. English and in Portuguese, the language in which our experiments were carried out:

1. Simple disjunctions (Either A or B), such as *Either it is raining or it is freezing: Ou está a chover ou está um gelo*.
2. Inclusive disjunctions (A or B, or both), such as *It is raining or it is freezing, or both: Está a chover ou está um gelo, ou ambos*.
3. Exclusive disjunctions (Either A or B, but not both), such as *Either it is raining or it is freezing, but not both*: *Ou está a chover ou está um gelo, mas não ambos.*

An exclusive disjunction differs from an inclusive one in that the former excludes the possibility in which both of its clauses hold, whereas the latter does not. We described the mental models and the fully explicit models for inclusive disjunctions earlier. Exclusive disjunctions are similar, except that a model of the joint occurrence of the two clauses represents an impossibility. Table 1 summarizes the fully explicit models for true and false inclusive disjunctions and for true and false exclusive disjunctions. A simple disjunction is in principle open to either an inclusive or an exclusive interpretation, but modulation as described in the next section can in some cases disambiguate it.

Possibilities call for logical analyses in modal logics, which concern various distinct interpretations of 'possible' and 'necessary' (see, e.g., Girle, 2009). But, in standard modal logics, an exclusive disjunction validly implies an inclusive disjunction of the same two clauses: In every case in which the exclusive disjunction is true, the inclusive disjunction is true too; that is, both disjunctions are true given that one of their clauses is true. But, in the mental model theory, the inference is not valid, because nothing to which a true exclusive disjunction refers implies the possibility of *A and B* (see Table 1). In fact, individuals reject the inference from an exclusive to an inclusive disjunction (Hinterecker et al., 2016).

Table 1
The conjunctions of exhaustive possibilities to which inclusive and exclusive disjunctions refer by default depending on whether they are true or false

Two Sorts of Disjunction	The Disjunction is True	The Disjunction is False
A or B, or both (inclusive)	A B	
	A ¬ B	
	¬ A B	
		¬ A ¬ B
A or B, not both (exclusive)		A B
	A ¬ B	
	¬ A B	
		¬ A ¬ B

Note: The remaining cases in each of the four cells denote impossible cases. The symbol '¬' denotes the negation of a clause.

1.3. Modulation and analytic truth values

Modulation is a process in the mental model theory in which the meanings of clauses, and knowledge of the context or of the world, can prevent the construction of models of possibilities, and add temporal, spatial, and other relations to models (Johnson-Laird & Byrne, 2002). Evidence shows that its effects occur both for conditionals (e.g., Juhos, Quelhas, & Johnson-Laird, 2012; Quelhas, Johnson-Laird, & Juhos, 2010) and for disjunctions (Quelhas & Johnson-Laird, 2017). The disjunctions in Table 1 by assumption are not ones that modulation has affected. A simple disjunction such as the following:

This year is a leap year or it is a year with a late Easter

can in theory be interpreted as inclusive or exclusive. Modulation can ensure an exclusive interpretation for certain simple disjunctions by eliminating the possibility in which both clauses hold, e.g.:

This year is a leap year or it is a common year.

People know that the two propositions are mutually exclusive: it cannot be both a leap year and a common year. The theory postulates that such knowledge is represented in a way that is equivalent to fully explicit models, and so mSentential compares the following two sets of possibilities:

Unmodulated simple disjunction		*Knowledge*	
leap year	¬ common year	leap year	¬ common year
¬ leap year	common year	¬ leap year	common year
leap year	common year		

Modulation uses the possibilities in knowledge to prevent the construction of a model of the possibility in which both clauses hold. No contradiction occurs because the possibility holds in default of information to the contrary, and knowledge has provided such information. The modulation establishes that the disjunction has the same possibilities as an exclusive one (see Table 1).

A consequence of modulation was not obvious until we had implemented it in the mSentential program. When an assertion has the same models as those in knowledge, it should be judged to be true. The standard term for assertions that are true in virtue of their meanings or knowledge is that they are *analytic*, though one influential philosopher has doubted their existence (see Quine, 1953). Knowledge, however, does not normally represent what is false. So the evaluation that a compound assertion is false in virtue of its meaning depends on determining that it is inconsistent with knowledge—a process that calls for deliberation, as does the judgment that it is contingent.

The disjunction:

Either it is a leap year or else it is a common year, but not both

should be judged to be true in virtue of its meaning. It illustrates one way in which analytic judgments should occur: The models of an assertion match those in knowledge. Another way is when models of individual clauses match those in knowledge. Consider, for instance, the exclusive disjunction:

Lisbon is in Portugal or Madrid is in Spain, but not both.

Both clauses are true, and so the exclusive disjunction is false. But, it is odd to assert a disjunction when both its clauses are true (cf. Grice, 1989). To avoid confusing our participants with such infelicities, we relied for the most part on the first way to establish truth-values for disjunctions. It uses the relation between the two propositions that their clauses assert. For example, knowledge of sighted persons establishes the possibilities that match those for the inclusive disjunction:

The sighted woman sees from her left eye or she sees from her right eye, or from both.
The falsity of the one proposition implies that the other one holds. So, even though it is not clear which possibility applies to the woman, the disjunction should be evaluated as true. Likewise, given that a blind person cannot see from either eye, this inclusive disjunction:

The blind woman sees from her left eye or she sees from her right eye, or from both is false. Knowledge also establishes the truth of certain exclusive disjunctions, such as:

Either the playing card is in a red suit or else it is in a black suit, but not both and the falsity of other exclusive disjunctions, such as in the following:

Either each week contains Tuesday or else each week contains Friday, but not both.

The task of evaluating each of the four cases in a disjunction's partition as possible or impossible should be straightforward for true disjunctions. But it should be much harder for false disjunctions, because individuals do not have access to the conditions in which disjunctions are false. They have to infer them, and the task depends on the deliberative system. For instance, suppose you have to enumerate the possibilities for the following disjunction given that it is known to be false:

It is raining or it is freezing, but not both.

Like the rest of us, you do not have these possibilities available to you at once. You need to infer them (see, e.g., Barres & Johnson-Laird, 2003; Khemlani, Orenes, & Johnson-Laird, 2014). The deliberation should proceed in the following way. You retrieve the possibilities for the disjunction to be true. One such possibility is that it is raining and not freezing, and so for the disjunction to be false, this possibility cannot hold. Another possibility for the disjunction to be true is that it is not raining but freezing, and so for the disjunction to be false, this possibility cannot hold, either. There are no other possibilities to which the true disjunction refers. So, for the disjunction to be false, these two cases need to be eliminated, and whatever possibilities remain are those for its falsity: it is possible that it is raining and freezing, and it is possible that it is not raining and not freezing (see Table 1). The fewer the models that deliberation has to infer, the easier the task should be. Hence, it should be easier for false inclusive disjunctions (one model) than for false exclusive disjunctions (two models; see Khemlani et al., 2014).

A common source of error in evaluating the possibilities for a false disjunction is to negate each element separately in a possibility for the true disjunction (Barres & Johnson-Laird, 2003; Santamaría & Espino, 2000). The possibilities for a true exclusive disjunction, *Either A or else B, but not both*, are as Table 1 shows:

A ¬B

¬A B

The negation of each element seperately in the first possibility yields:

¬A B

and the negation of each element separately in the second possibility is as follows:

A ¬B.

So, this *mirror-image* error yields the identical the possibilities to those for a true disjunction rather than the correct possibilities for a false disjunction.

Previous studies of the mental model theory have shown that disjunctions can yield predicted illusory inferences that seem valid but that are in fact invalid (see Khemlani & Johnson-Laird, 2017). Could there also be illusions of truth? The extension of modulation to analytic disjunctions yields the following recipe. Take two predicates that are known to be mutually exclusive, and apply them to the same subject in an exclusive disjunction. If the two predicates are not exhaustive, then the resulting assertion should yield an illusion of truth, as in our opening example:

Either the food is hot or else it is tepid, but not both.

The two predicates leave open the possibility that neither of them holds, for example,the food is cold, and so the disjunction could be false. However, the models of the disjunction represent only the two alternative possibilities for the food, and not the possibility that it is cold. Hence, participants should tend to match their models of the disjunction to two alternatives in their knowledge of the temperatures of food, and therefore they should judge that the disjunction is true.

In sum, the mental model theory yields three principal predictions:

1. Participants should judge that certain disjunctions are true, that certain disjunctions are false, and that certain disjunctions are contingent. And they should succumb to illusions of truth.
2. They should evaluate the predicted possibilities and impossibilities for inclusive and exclusive disjunctions.

3. They should tend to carry out both the two preceding tasks more accurately for true disjunctions, which can rely on immediate access to knowledge, than for false disjunctions, which call for deliberative inferences. They should often err for false exclusive disjunctions by making the mirror-image error in which they accept the same possibilities that hold for true disjunctions.

We report three experiments that tested these predictions.

2. Experiment 1: Simple disjunctions

This experiment examined the participants' judgments of the truth-values of simple disjunctions as true, false, or impossible to determine (without further information), and their evaluations of the cases in the disjunctions' partitions as possible or impossible. The three sorts of judgment correspond to three sorts of simple disjunction used in the experiment:

1. Those for which modulation should yield a true disjunction, such as in the following:

   Either this year is leap year or it is a common year.
   (Ou este ano é bissexto ou então é um ano comum.)

Participants should judge that this assertion is true.

2. Those for which modulation should yield a false disjunction, such as:

   Either it is March or it is the third month of the year.
   (Ou é o mês de Março ou então é o terceiro mês do ano civil.)

Participants should judge that this assertion is false. The Portuguese structure of these disjunctions, 'Ou … ou …,' is analogous to an exclusive disjunction, and so the example should be judged as false even if the experiment had been carried out in March. Both its clauses refer to the same month.

3. Those for which modulation should have no effect, so they should yield a contingent disjunction, such as:

   Either it is a holiday or it is a Tuesday.
   (Ou é dia Feriado ou então é Terça-feira.)

The pronoun 'it' does not occur in Portuguese, which uses only the present tense akin to 'is' referring to the present time. Participants should judge that it is impossible to determine whether this disjunction is true or false—unless by chance they were tested on a Tuesday. They would not have been tested on a holiday.

Table 1 above shows the predicted evaluations of possibility and impossibility for the cases in the partitions for the true and false disjunctions. Contingent disjunctions should yield a set in which all four cases in the partition are possible. This experiment and the following ones were carried out in Portuguese, the native language of the participants, and so in what follows, we have translated the materials and instructions into English.

2.1. Method

2.1.1. Participants

The experiment tested 40 psychology students from ISPA's laboratory pool (14 male and 26 female, with mean age of 23.1 years). They participated in exchange for partial fulfilment of a course requirement.

2.1.2. Design

The participants carried the two tasks in a counterbalanced order on four disjunctions of each of three predicted sorts: true, false, and contingent. One task was to judge each disjunction as 'true', 'false', or 'impossible to determine'. The other task was to evaluate each of the four cases in a disjunction's partition as 'possible' or 'impossible'. For this task, each participant received half of the problems with the partition in the following order: *A and B*, *A and not-B*, *not-A and B*, *not-A and not-B*, and the other half of the problems in the reverse order. The two tasks were carried out in separate blocks, each using a different random order of disjunctions for each participant, and the order of the two blocks was counterbalanced over two separate groups of participants.

2.1.3. Materials

Each disjunction was of a simple sort: *Either A or B* (*Ou A ou B* in Portuguese) where the subjects of the two clauses *A* and *B* had the same referent. We devised the three sorts of disjunction based on everyday topics, such as books, the calendar, and animals. For true disjunctions, we used two mutually exclusive and exhaustive categories for the subject of the clauses, for example:

Either the book is brand new or it is used.

For false disjunctions, we selected two equivalent categories for the subject, e.g.:

Either the book is a cookbook or it is a recipe book.

And for contingent disjunctions, we selected two categories that were independent of one another for the subject, for example:

Either the book is poems or it is in Portuguese.

The 12 disjunctions are in Appendix A of the Supplemental Material file.

2.1.4. Procedure

The experiment was carried out using the E-prime program, and the participants sat in individual cubicles in front of computers to make their responses. The first screen presented the instructions, for which the key part was as follows:

This is neither an intelligence test nor a personality test. It is concerned only with how people evaluate whether an assertion is true or false, and the situations that they consider to be possible or impossible given the same assertion. Therefore, we ask you to be careful in reading the assertions, which will always appear ***in bold****, and to be careful in answering every question about these assertions.*

The disjunctions were always about specific entities, such as 'the book,' 'this year', 'the dog', which were presented with no preceding context. So, to help the participants become comfortable with definite descriptions of the sort, and with the two tasks, the instructions included a practice trial for both sorts of task with a similar sentence, which was not used in the experiment proper. The participants were told that there was no time limit on completing the experiment.

Each task was presented on a separate screen. Here is an English translation of a screen shot of a trial calling for the judgment of the truth-value of a disjunction:

Please consider the following sentence:

Either the letter is a vowel or it is a consonant.

Do you consider that this sentence is:

True **(1)** False **(2)** Impossible to determine **(3)**

(Press the key corresponding to the number of your choice)

Each case in the partition was then presented on a separate screen for the participants to evaluate as 'possible' or 'impossible', for example:

Please consider the following sentence:

Either the letter is a vowel or it is a consonant.

Given the statement above, is the following situation:

The letter is a vowel and it is a consonant. Possible **(1)** Impossible **(2)**

(Press the key corresponding to the number of your choice)

2.2. Results

The two groups for counterbalancing the order of the tasks did not differ reliably in the percentages of predicted judgments of the truth-values of the disjunctions, or in the percentages of predicted evaluations of the cases in their partitions (Mann–Whitney tests, $z < 1$, $p > .3$), and so we amalgamated the results for analysis. Table 2 presents the percentages of the judgments of the truth-values of the disjunctions. The results for the individual disjunctions are in the Supplemental Material File (Tables 1, 2, and 3). All the participants judged some disjunctions to be true, some disjunctions to be false, and some

Table 2
The percentages of judgments in Experiment 1 for the three sorts of simple disjunction (modulated to be true or false, or contingent) as 'True', 'False', or 'Impossible to determine', with predicted evaluations in bold

	The Three Sorts of Judgment		
Three Sorts of Disjunction	'True'	'False'	'Impossible to Determine'
True	**84**	9	7
False	43	**47**	10
Contingent	20	37	**43**

disjunctions to be contingent. As the mental model theory predicts, the true disjunctions yielded more predicted judgments of truth-value (84%) than the false ones (47%), and the difference was reliable (Wilcoxon test, $z = 4.27$, $p < .001$). Indeed, participants concurred with the predictions for true disjunctions (Wilcoxon test, $z = 4.93$, $p < .001$), but not reliably for false disjunctions (Wilcoxon test, $z < 1$, $p > .5$) or for contingent ones (Wilcoxon test, $z = 1.15$, $p > .25$). As Table 2 shows, the most frequent error for false disjunctions was to judge them to be true (43%), and the most frequent error for contingent disjunctions was to judge them to be false (37%).

Table 3 presents the evaluation of the cases in the disjunctions' partitions as 'possible' or 'impossible'. All 16 sorts of evaluation, including the selection of no cases as possible, occurred at least once in the experiment, and so the chance probability of a selection is 1/16.

The table shows the sets of evaluations that occurred on at least 5% of trials, and the balance of percentages in each row is the remaining miscellaneous evaluations. The set of evaluations for the true disjunctions occurred reliably (27 out of the 40 participants made this set of evaluations more often than not, 11 made another sort of evaluation, and there were two ties, Binomial test, $p < .001$). The set of evaluations for false disjunctions was most often the one for true disjunctions (34%), but 27 participants made some other sort of evaluation more often than not, only 5 made the sort for false disjunctions, and the rest were ties, so the bias was for some other sort (Binomial test, $p < .001$). In part,

Table 3
The percentages of sets of evaluations in Experiment 1 for each case in the partitions of the three sorts of disjunction. The cases shown in a set are those that participants evaluated as possible, and the cases not shown in a set are those that they evaluated as impossible. The balances of percentages in each row are miscellaneous sets of evaluations that each occurred on less than 5% of trials. The predicted sets are those with the percentages in bold, and the symbol '¬' denotes the negation of a clause

	The Sets of Evaluations as 'Possible' of the Cases in the Partitions of Simple Disjunctions			
Three sorts of predicted disjunction	A ¬ B ¬A B	A B ¬A ¬ B	A B A ¬ B ¬ A B ¬ A ¬ B	A B
True	**71**	0	2	2
False	34	**28**	3	15
Contingent	43	0	**42**	0

the false disjunctions might have elicited the evaluation for true disjunctions, because the participants judged the disjunction itself as true (see Table 2). But, this explanation cannot be the whole story, because the error occurred on 41% of all those trials in which the participants judged any sort of disjunction as false. The error therefore corroborates the predicted mirror image selection of possibilities (section 1.3.), which yields a true set of evaluations for false disjunctions. A frequent evaluation for contingent disjunctions was the predicted one, and 17 out of the 40 participants yielded this set more often than any other one, and there was one tie (Binomial, $p < .001$). However, many participants evaluated the possibilities as if the disjunction was true (43% in Table 3).

2.3. Discussion

The results corroborated the prediction that individuals should judge certain disjunctions to be true, others to be false, and still others to be impossible to determine. They also corroborated the prediction that the task should be easier for true disjunctions than for false disjunctions—indeed, the results showed that the participants were quite likely to evaluate possibilities for false disjunctions as though they were true. This mirror image error, as the theory predicts, appears to be a consequence of constructing negations of individual clauses, which yields the same set of possibilities for false disjunctions as for true disjunctions. Disjunctions predicted to be false were often judged to be true (43% in Table 2), for example, the disjunction: ‘Either it’s March or else it’s the third month of the civil year.’ However, a simple disjunction can be interpreted as inclusive, and such disjunctions are true, because they allow both of their clauses to be true. So, this judgment is in accord with the theory.

One surprising and unpredicted result is that disjunctions predicted to be contingent were often judged to be false (37% in Table 2). The mental model theory suggests that a contingent disjunction, such as:

Either the letter is in Times New Roman or it is a capital letter

is judged to be false (58%; Table 3 in the Supplemental Material file), because it elicits an illusion of falsity. Individuals know that letters can be neither in Times New Roman nor in capitals, and so they evaluate the disjunction as, ‘False’. One possibility is that they overlook that the disjunction concerns a particular letter, and it could be true or false about this letter, so in fact it is contingent. We therefore looked out for such illusions in our subsequent study of exclusive disjunctions. The ambiguity of simple disjunctions was a factor in present study, and so we turned to unambiguous disjunctions in the next experiment and avoided contingent disjunctions.

3. Experiment 2: Inclusive disjunctions

This experiment used the same two tasks as the previous one, but it examined unambiguous inclusive disjunctions: *A or B, or both*. They were modulated to be either true or false, and the participants evaluated them as true, false, or impossible to determine, and judged the possibility or impossibility of each case in their partitions. An example of an inclusive disjunction that should tend to be judged as true is as follows:

The profitable shop has good products or it has big promotions, or both.

The disjunctions cannot be tautologies, such as 'The number is odd or it is not odd', because naive individuals would reject them as inclusive disjunctions—a number cannot be both odd and not odd. So the contents were designed to elicit judgments of truth, and the rejection as impossible those cases in the partition corresponding to both clauses as false. The mental model theory predicts that individuals should judge each case in its partition as possible except for the *not-A and not-B* case in which the shop has neither good products nor big promotions (see Table 1). The experiment also used the same contents with a negated subject:

The unprofitable shop has good products or it has big promotions, or both.

The mental model theory predicts that it should be judged to be false because it is impossible for an unprofitable shop to have good products (*A*) or big promotions (*B*), let alone both. So, only one case is possible: *not-A and not-B*. But another interpretation of false inclusive disjunctions is feasible. The disjunction is false, because the joint occurrence of *A and B* is impossible, e.g., of both good products and big promotions. So, three cases are possible: *A and not-B*, *not-A and B*, and *not-A and not B*. There were therefore two predicted sets of possibilities for the false inclusive disjunctions: the set in Table 1 with just one possibility, and this new set with three possibilities.

3.1. Method

3.1.1. Participants

The experiment tested 81 psychology undergraduates from the same population as before (71 female and 10 male, with a mean age of 19.8 years), who volunteered to take part in the experiment.

3.1.2. Design

The participants carried out the two tasks in separate blocks of trials with six inclusive disjunctions. One task was to judge each disjunction as true, false, or impossible to determine. The other task was to evaluate each of the cases in the six disjunctions' partitions as 'possible' or 'impossible'. As the true and false disjunctions were matched pairs, we constructed two sets of materials to avoid the participants having to deal with both members of each pair, and assigned each participant at random to one of the two sets. We used two different orders of the cases in the partition (as in Experiment 1). The two tasks were carried out in separate blocks, each using a different random order of disjunctions for each participant, and the order of the two blocks was counterbalanced using two separate groups of participants.

3.1.3. Materials and procedure

We devised six true and six false inclusive disjunctions of the sort, *A or B, or both*, about everyday matters. The true disjunctions had two predicates that were exhaustive but that could both hold for the subject of the sentence, for example:

The sighted woman sees properly from her left eye or she sees properly from her right eye, or both.

The false disjunctions used the same predicates but ascribed them to the negation of the subject, for example:

The blind woman sees properly from her left eye or she sees properly from her right eye, or both.

The complete set of disjunctions is in Appendix C of the Supplemental Material file.

The participants were tested in small groups, and the problems were presented to them in booklets with a separate page for instructions for each block of trials. The four cases in the partition were presented on a single sheet in the booklet. Otherwise, the procedure was similar to that for Experiment 1.

3.2. Results and discussion

The two groups for counterbalancing the order of the two tasks did not differ reliably in the percentages of predicted judgments of the truth-values of the disjunctions, or in the percentages of predicted evaluations of the cases in the partitions (Mann–Whitney tests, $z < 1$, $p > .6$), and so we amalgamated the results for analysis. Table 4 presents the percentages of the judgment for the true and false disjunctions. The participants made reliably more predicted judgments (65%) than unpredicted judgments (35%; Wilcoxon test, $z = 4.53$, $p < .001$). The predicted judgments occurred reliably for true disjunctions (Wilcoxon test, $z = 5.06$, $p < .001$) and for false disjunctions (Wilcoxon test, $z = 2.06$, $p < .025$). But, as the theory predicts, the true disjunctions yielded more predicted judgments than the false disjunctions did (Wilcoxon test, $z = 3.38$, $p < .001$). The proportions of judgments opposite in truth-value to the predictions were small in this experiment, perhaps because the disjunctions were clearly inclusive.

Table 5 presents the sets of evaluations of the four cases in the disjunctions' partitions that occurred on at least 5% of trials. The evaluations of the true disjunctions

Table 4
The percentages of judgments in Experiment 2 for the two sorts of inclusive disjunction (modulated to be true and modulated to be false) as 'True', 'False', or 'Impossible to determine', with predicted evaluations in bold

	The Three Sorts of Judgment		
Two Sorts of Inclusive Disjunction	'True'	'False'	'Impossible to determine'
True	**72**	7	21
False	12	**58**	30

Table 5
The percentages of sets of evaluations in Experiment 2 for the cases in the partitions of the inclusive disjunctions. The cases shown in a set are those that participants judged to be possible, and the cases not shown in a set are those that they judged to be impossible. The balances of percentages in each row are miscellaneous sets of evaluations that occurred on less than 5% of trials. The predicted sets are those with the percentages in bold, and the symbol '¬' denotes the negation of a clause

	The Sets of Evaluations of the Cases in the Partitions of Disjunctions			
Two sorts of inclusive disjunction	A B A ¬ B ¬ A B	¬ A ¬ B	A ¬ B ¬ A B ¬ A ¬ B	A B
True	**74**	0	1	10
False	42	**14**	**17**	5

corroborated the predicted set: 63 out of the 81 participants made these evaluations more often than others, and 18 did not (Binomial test, $p < .001$). The two predicted sets of evaluations for false inclusive disjunctions (a single possibility and three possibilities in bold in Table 5) also occurred reliably: 20 out of the 81 participants made one or other of these sets of evaluation more often than others (Binomial test, $p < .003$).

Once again, the participants judged some disjunctions as true and some as false. The task was easier for true disjunctions than for false disjunctions. The sets of the participants' evaluations of the cases in the partitions likewise corroborated the prediction for true disjunctions, but as in the previous study the participants were quite likely to make evaluations for the false disjunctions that matched those for true disjunctions, and this set occurred at a similar rate (43%) for any sort of disjunction that the participants judged to be 'false'. So, as in the previous experiment, the error supports the mirror-image error (see section 1.3).

4. Experiment 3: Exclusive disjunctions

The grammatical cue to an exclusive disjunction is the tag, *but not both* (*mas não ambos*), and this experiment used this cue. It examined disjunctions modulated to be true, such as:

1. True: Either the card is in a red suit or it is in a black suit, but not both.

The two predicates are exhaustive, and so the disjunction is true. The experiment examined a matching false disjunction:

2. False, but matching true: Either the hearts suit contains a queen or else it contains a knave, but not both.

The hearts suit contains both cards, and so the disjunction is false (see Table 1 for the partitions of these two disjunctions). The experiment examined illusorily true disjunctions, such as:

3. Illusorily true: Either it is Christmas or else it is April, but not both.

The two predicates are mutually exclusive but not exhaustive, and so they should elicit an illusory judgment of truth. The evaluation of the partition may therefore exclude the possibility of neither Christmas nor April. But, it could also elicit this possibility. So there are two sets of possibilities for illusorily true disjunctions, *Either A or else B, but not both*, namely:

A ¬ B

¬ A B

and:

A ¬ B

¬ A B

¬ A ¬ B

A corollary is that the matching false disjunctions also have two sets of possibilities: the remaining cases in the partition for the two sets above. For example, the following disjunction should be judged to be false, and elicit both sets:
4. False, but matching illusorily true: Either each week contains Tuesday or else each week contains Friday, but not both.

Finally, the experiment examined contingent disjunctions on which modulation has no effect, and so all four cases in the partition should be possible, e.g.:
5. Contingent: Either the meal contains fish or it contains vegetables, but not both.
However, these disjunctions should also elicit illusions of falsity, which we discussed in relation to Experiment 1.

4.1. Method

4.1.1. Participants

The experiment tested 73 psychology undergraduates from the same population as before (61 female and 12 male, with a mean age of 19.0 years), who volunteered to take part in the experiment.

4.1.2. Design

The participants carried out the same two tasks as in the previous experiments: the judgment of each exclusive disjunction as true, false, or impossible to determine; and the evaluation of the four cases in a disjunction's partition as 'possible' or 'impossible'. The experiment examined a total of 12 disjunctions: four disjunctions that should be judged as true (two illusory and two controls), four that should be judged as false (corresponding to the two sorts of true ones), and four that should be judged as impossible to determine. We used two different orders of the cases in the partition (as in Experiment 1). The two tasks were carried out in separate blocks, each using a different random order of disjunctions for each participant, and the order of the two blocks was counterbalanced using two separate groups of participants.

4.1.3. Materials and procedure

The experiment was based on 12 exclusive disjunctions of the sort: *Either A or B, but not both*. Their contents concerned the seasons and the calendar, food, playing cards, and health. We constructed the true and false disjunctions as in Experiment 1. For true disjunctions, we used two mutual exclusive but exhaustive categories for the subject of the two clauses, e.g.:
Either the girl is healthy or she is ill, but not both.
For false disjunctions, we selected two equivalent categories for the subject of the two clauses,for example:
Either the boy has fever or he has a temperature of at least 37°, but not both.

For the illusorily true disjunctions, we selected alternative categories that were not exhaustive for the subject of the two clauses, for example:
Either the food is cold or it is tepid, but not both.

And for contingent disjunctions, we selected two categories that were independent of one another, e.g.:
Either it is Spring or else the sun is shining today, but not both.
The 12 disjunctions are in Appendix E of the Supplemental Material file. The procedure and instructions were identical to those of Experiment 2.

4.2. Results and discussion

The two groups for counterbalancing the order of the two tasks did not differ reliably in the percentages of their predicted judgments of the truth-values of the disjunction assertions (Mann–Whitney test, $z < 1$, $p > .3$), and so we amalgamated these results for statistical analysis. But, the two groups did yield a reliable difference in the percentages of predicted evaluations

Table 6
The percentages of judgments in Experiment 3 of the five sorts of inclusive disjunction (true, false but matching true, illusorily true, false but matching illusory true, and contingent) as ‘True’, ‘False’, or ‘Impossible to determine’, with predicted evaluations in bold, and asterisks indicate that Wilcoxon tests showed a reliable bias for the judgment (z values > 4.6, p values < .001). The second row in the table sums only to 99% because of rounding

	The Three Sorts of Judgment		
Five Sorts of ExclusiveDisjunction	‘True’	‘False’	‘Impossible to determine’
True	**78***	10	12
False, but matching true	14	**71***	14
Illusorily true	**88***	8	4
False, but matching illusorily true	17	**76***	7
Contingent	5	69	**26**

of the partitions (Mann–Whitney, $z = 3.5$, $p < .001$, two tail), and so we kept their data separate for analysis.

Table 6 presents the percentages of predicted judgments of truth-values for the five sorts of exclusive disjunction. As it shows, the four sorts of disjunction with predicted truth-values corroborated the predictions reliably, including compelling illusions of truth (88%), and the frequencies of judgments of truth-values opposite to the predicted ones were small. The contingent disjunctions gave rise to illusions of falsity to a high degree (69%; Wilcoxon, z = 0.3.78, $p < .001$, two tail). This finding corroborates the same tendency in Experiment 1. It lends credence to our post hoc hypothesis that individuals judge that a disjunction is false in general, and so conclude that it is false about a specific case.

Table 7 presents the evaluations as 'possible' or 'impossible' for each of the four cases in the partitions of the five sorts of disjunction. Because the order of the two tasks had a reliable effect on the frequencies of predicted sets, the table shows separate percentages for the two groups of participants that counterbalanced this order: Group I judged truth values first, and Group II evaluated possibilities for the partitions first. As the first column of results in Table 7 shows, when the participants evaluated the possibilities in the partitions first (II), their evaluations were more likely to be those for true exclusive disjunctions than when they judged truth-values first (I) (Mann–Whitney test, $z = 3.8$, $p < .001$, two tail). The effect even included an increase in judgments of truth of an illusory sort. And the only disjunctions

Table 7

The percentages of sets of evaluations in Experiment 3 for the cases in the partitions of the exclusive disjunctions: Group I judged truth-values first, and Group II evaluated possibilities for the partitions first. The illusorily true disjunctions and their matching false controls have two predicted sets of evaluations. The cases shown in a set are those that participants judged to be possible, and the cases not shown in a set are those that they judged to be impossible. The balances of percentages in each row are miscellaneous sets of judgment that occurred on less than 5% of trials. The predicted sets are those with the percentages in bold, and the symbol '¬' denotes the negation of a clause

		The Sets of Evaluations of the Cases in the Partitions of Disjunctions				
Five sorts of disjunction	Two groups	A ¬ B ¬A B	A B ¬A ¬ B	A ¬ B ¬ A B ¬ A ¬ B	A B	A B A ¬ B ¬ A B ¬ A ¬ B
True	I. Truth-values first	**66**	0	16	1	5
	II. Possibilities first	**74**	0	19	0	3
False, but matching true	I. Truth-values first	24	**17**	16	4	7
	II. Possibilities first	51	**9**	16	6	1
Illusorily true	I. Truth-values first	**25**	0	**59**	0	4
	II. Possibilities first	**64**	0	**33**	0	1
False, but matching illusorily true	I. Truth-values first	29	**1**	17	**33**	3
	II. Possibilities first	56	**1**	11	**14**	3
Contingent	I. Truth-values first	20	1	18	0	**47**
	II. Possibilities first	67	0	19	0	**11**

that did not show this increase reliably were the true disjunctions (Mann–Whitney test, $z = 1.26$, $p > .2$), but they had high percentages of judgments of truth in both groups. For the other four sorts of disjunction the increase was reliable (Mann–Whitney tests, z was at least 2.48, $p < .015$, two tail). The corollary is that when the participants judged truth-values first (I), they tended to evaluate partitions in ways other than for true disjunctions. In both groups, as predicted, true disjunctions including illusory ones (58%) yielded more predicted evaluations of possibilities than false disjunctions (19%; Wilcoxon test, $z = 6.87$, $p < .001$). As the mirror image error predicts, the participants tended to evaluate false disjunctions as having the same possibilities as true disjunctions.

Overall, the participants judged that some exclusive disjunctions were true, that some were false, and that some were contingent. They also made the predicted illusory judgments of truth. Their judgments of truth-value and their evaluations of the possibilities in partitions were more accurate—illusions apart—for true disjunctions than for false disjunctions. And for the false disjunctions, they often made the mirror-image error of accepting the possibilities for true disjunctions (24% for the group who judged truth-value first and 51% for the group who evaluated possibilities first, see Table 7). These results corroborated the mental model theory's predictions. There was one unpredicted result: the order of the two tasks had a marked effect on the evaluation of the possibilities in partitions. The participants' evaluations were more likely to be those for true disjunctions when they evaluated possibilities first than when they judged truth-values first. A contributory factor here, which the mental model theory suggests, is once again the role of the mirror-image errors in evaluations of the partitions for false disjunctions.

5. General discussion

Our experiments corroborated the models theory's three main predictions about disjunctions.

Prediction 1: when a disjunction refers to the same possibilities as those in knowledge, participants tended to judge that the disjunction was true; when it refers to possibilities that conflict with those in knowledge, they tended to judge that it was false; and otherwise, they tended to judge that is impossible to determine whether it is true or false, that is, it is contingent. These judgments of truth-value showed that the existence of assertions that are true (and that are false) on the basis of their meanings or knowledge is no longer an unempirical dogma (contrary to Quine, 1953), and that it is no longer restricted to the special case of conditionals (Quelhas et al., 2017), because it occurs with disjunctions too. However, as predicted, true judgments included illusory ones for disjunctions that in fact are contingent. They occured when two predicates in an exclusive disjunction were mutually exclusive but not exhaustive, 'Either it is Christmas or else it is April, but not both'. Mental models of the disjunction represent only those clauses in compounds that are true in a possibility.

Christmas

April

Which match models in participants' knowledge of distinct periods of the year, and so they judge that the disjunction is true. It is an illusion, because it overlooks that knowledge allows other possibilities in which both clauses in the disjunction are false, for example, October—a possibility that is not represented explicitly in the models of the disjunction. So the disjunction is in reality contingent: It can be true or false. Our results corroborated the occurrence of such illusions of truth (Table 7 for Experiment 3).

Prediction 2: a crucial result is that participants inferred the theory's predicted cases in the disjunctions' partitions as possible or impossible. Simple true disjunctions, such as 'Either this year is a leap year or it is a common year' elicited evaluations of possibilities corresponding to an exclusive interpretation (71% in Table 3). True inclusive disjunctions (including 'or both') elicited evaluations of the three possibilities corresponding to an inclusive interpretation (74% in Table 5). And true exclusive disjunctions ('but not both') elicited evaluations of the two possibilities corresponding to an exclusive interpretation (66% when the participants judged truth values first, and 74% when they evaluated possibilities first, see Table 7).

Prediction 3: the participants carried out both tasks more accurately for true disjunctions than for false disjunctions. Individuals do not know the cases in which disjunctions are false, and have to infer them from the cases in which disjunctions are true. The fewer the models they have to infer, the easier the task should be; for example, it should be easier to infer the one possibility in which an inclusive disjunction is false (71% in Table 6), than the two possibilities in which an exclusive disjunction is false (58% in Table 4). Likewise, evaluations of the possibilities in partitions for false disjunctions often corresponded to the partitions for true disjunctions (34% in Table 3 for simple disjunctions; 42% in Table 5 for inclusive disjunctions, and 24% and 51% for the two groups in Table 7 for exclusive disjunction). The mirror-image error predicts these results. Individuals negate each element in the true possibilities instead of inferring the complement of the true possibilities (see Section 1.3).

One major unexpected result occurred: the participants often judged that contingent disjunctions were false in the two experiments that included them (37% in Experiment 1; 69% in Experiment 3). Our conjecture after Experiment 1 was that this error with exclusive disjunctions was an illusion of falsity, which occurred because the disjunction is false as a general claim, and so individuals judge it to be false, overlooking that it could be true as a claim about a specific case. For instance, given the contingent disjunction:

Either the meal contains fish or else it contains vegetables, but not both.

Individuals infer that it is false for many meals, so they judge it to be false. But, it could be true for the meal in question, just as it could be false. So the correct judgment is that the disjunction is contingent. Nonetheless, the evaluation of these disjunctions' partitions hardly ever corresponded to a false exclusive disjunction (0% in Table 3 for Experiment 1, and 1% in Table 7 for Experiment 3). The evaluations of contingent disjunctions tended to be either correct or else to be those for true disjunctions (see Tables 3 and 7). So the evaluation of the partition is likely to include mirror-image errors.

Overall, the results supported the mental model theory. Knowledge modulates the models of disjunctions. They are the basis of judgments of truth-value, the evaluations of sets of possibilities, and the illusions of truth. Individuals lack an immediate access to the possibilities to which false disjunctions refer, and so they are liable to make a mirror-image error in assessing them. Of course, the findings could be open to an alternative explanation, and so we consider in turn the feasibility of theories based on logic, pragmatics, or probability.

In classical logic, a disjunction is true if at least one of its clauses is true. If both of them are true, then an inclusive disjunction is true, but an exclusive disjunction is false. But, psychological theories based on logic (e.g., Braine, 1978; Rips, 1994) fail to explain two of our principal results. First, they offer no explanation for disjunctions yielding illusory truth values. Second, they cannot make any predictions about possibilities, because they are based on classical predicate logic, which excludes them. Modal logics deal with possibilities (see, e.g., Girle, 2009), but they cannot account for results, either. Consider, for instance, the following disjunction:

Either the card is in a red suit or it is in a black suit, but not both.

It yielded an evaluation that the following case was possible:

The card is in a red suit and it is not in a black suit.

In any standard modal logic, the inference is invalid. Suppose it is impossible that the card is in a red suit, because it is in a black suit; the disjunction is true but the case above is impossible. In our experiments, all the evaluations of cases in the disjunctions' partitions as 'possible' are invalid in modal logics. If these evaluations are based on modal logics, then our participants should at least have balked at the task. They did not.

One way in which to try to defend theories based on classical logic is to argue that evaluations of possibilities rely on a pragmatics of a sort that Grice (1989) proposed. His hypothesis is that speakers communicate more than they say. For instance, an assertion such as in the following:

It is raining or it is freezing, or both yields a 'conversational implicature' that the speaker does not know which of the two holds. If the speaker had known, the utterance would have pin-pointed one of them. Such implicatures are not valid deductions, and so speakers can cancel them without contradiction, as in:

It is raining or it is freezing, or both, and I know which, but am not telling you.

Inferences from disjunctions to inferences of possibilities might also be conversational implicatures (see, e.g., Sauerland, 2004):

It is raining or it is freezing, or both.
So it is possible that it is raining.
So it is possible that is freezing.

Conversational implicatures, however, can be denied without self-contraction (Grice, 1989). But consider the following assertion, which conjoins the disjunction with a denial of each of its alleged implicatures:

It is raining or it is freezing or both, and it is not possible that it is raining and it is not possible that it is freezing.

According to Grice (1989), implicatures can be denied without self-contradition. Yet the preceding denials yield a self-contradiction: The two assertions of what is not possible refute the disjunction. It follows that the two clauses that are denied cannot be conversational implicatures.

Grice (1989) postulated another sort of implicature, a 'conventional' one, which depends on the meanings of terms. His example was as follows:

He is British; therefore, he is brave.

The mental model theory proposes that the meaning of 'or' yield possibilities as defaults, just as the meaning of 'dog' yields, say, *having four legs* as a default. Perhaps this analysis is equivalent to treating the inferences as conventional implicatures, but it depends on a semantics for 'or' that differs from its meaning in logic. Either way, then, the logical treatment of these inferences fails.

As we mentioned in the Introduction, a recent approach to the psychology of reasoning bases it on probabilities (e.g., Adams, 1998; Cruz, Over, & Oaksford, 2017; Oaksford & Chater, 2007; Pfeifer, 2013). It replaces the notion of validity with probabilistic validity (p-validity), and an inference from a single premise to a conclusion is p-valid if for any consistent assignment of probabilities to its clauses, the conclusion cannot be less probable than the premise. Consider inferences from a disjunction to the three possibilities that it implies as defaults. The disjunction is:

The flaw is in the software or it is in the hardware, or both.

It implies the possibility of a flaw in one or both of the software and hardware (in default of knowledge to the contrary):

Therefore, it is possible that the flaw is in the software.
Therefore, it is possible that the flaw in the hardare.
Therefore, it is possible that the flaw is in the software and it is in the hardware.

Individuals draw all three of these conclusions (Hinterecker et al., 2016). Likewise, if you know that the three preceding possibilities are exhaustive, they imply the disjunction as a conclusion. So, a reasonable assumption is that the sum of their probabilities should equal the probability of the premise. Hence, given that individuals in an experiment drew each inference separately, each inference violates *p*-validity. Indeed, the participants tended to estimate the probabilities of each conclusion as smaller than the probability of the premise. Other disjunctive inferences, which are accepted in daily life, are striking counterexamples to the probabilistic approach,for example:

It is possible that Ben is in Manhattan or it is possible that he is in Princeton.

So it is possible that he is in Princeton.

The inference is valid in the mental model theory, because the conclusion is one of the conjunctions of possibilities to which the premise refers. Yet the inference violates p-validity, because the premise can be more probable than the conclusion, and it is bound to be more probable if both clauses have a probability greater than zero.

The mystery of disjunctions is, as we said in the Introduction, what makes them difficult. Neither classical logic nor probabilistic logic predicts their difficulty. The mental model theory offers an explanation: It is because they refer to two or three different possibilities (Johnson-Laird, 2006). These numbers may seem surprisingly small, but to grasp their effects, try the following inference from two inclusive disjunctions:

Ann is in Acton or Bill is in Boston.
Bill is not in Boston or Cate is in Cambridge.
What follows?

Logicians consider the inference obvious and embody it in a well-known rule of inference, the 'resolution' rule, for automatic theorem provers (e.g., Robinson, 1979). The rule yields another inclusive disjunction as its conclusion:

Ann is in Acton or Cate is in Cambridge, or both.

When naive individuals reason from such premises, they are more likely to err than not (e.g., Johnson-Laird et al., 1992). The reason is that each premise calls for models of distinct possibilities, and the inference depends on combining these models properly. The present studies have borne out the mental model theory's account, not only showing that individuals can use their knowledge to determine whether or not disjunctions are true, but also corroborating their evaluations of the possibilities for different sorts of disjunction. They are quite accurate for true disjunctions, but less often for false and contingent disjunctions. Their judgments of truth values and evaluations of partitions corroborate the mental model theory, but challenge theories based on logic or on probability.

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Supporting Information

Additional Supporting Information may be found online in the supporting information tab for this article:

Appendix S1. The 12 disjunctions used in Experiment 1 (in English and in the original Portuguese).

Appendix S2. The 12 disjunctions used in Experiment 1, with percentages on truth evaluations and on predicted possibilities.

Appendix S3. The 12 inclusive disjunctions, with the form ‘A or B, or both’, used in Experiment 2 (in English and in the original Portuguese).

Appendix S4. The 12 inclusive disjunctions used in Experiment 2, with percentages of truth evaluations and predicted answers in the four possibilities.

Appendix S5. The 12 exclusive disjunctions in the form ‘Either A or else B, but not both’ used in Experiment 3 (in English and in the original Portuguese).

Appendix S6. The 12 exclusive disjunctions in the form ‘Either A or else B, but not both’ used in Experiment 3, with percentages on truth evaluations and on predicted possibilities. The number in each disjunction correspond to the topic of the content: 1 – seasons/calendar; 2 - food; 3 –playing-card; 4 – health.